SIMPLE, CONTINUOUS AND CONSISTENT PHYSICS BASED MODEL FOR Flicker NOISE IN MOS TRANSISTORS

Alfredo Arnaud(1), Carlos Galup-Montoro(2)

(1) GME – IIE, Facultad de Ingeniería, Universidad de la República, Montevideo – Uruguay.
(2) LCI, Departamento de Engenharia Eletrica, Universidade Federal de Santa Catarina.

ABSTRACT

Although there is still controversy about its origin, the designer requires accurate models to estimate 1/f noise of the MOS transistor in terms of its size, bias point and technology. Conventional models present limitations; they usually do not consistently represent the series-parallel association of transistors and they may not provide adequate results for all the operation regions, particularly moderate inversion. In this work we review current flicker noise models, paying particular attention to their behavior along the different operation regions and to their series-parallel association properties. We present a physics based model for flicker noise following classical carrier fluctuation theory. With the aid of a compact, continuous model for the MOS transistor it has been possible to integrate the contribution to drain current noise of the whole channel area arriving at a consistent, continuous, and simple model for the 1/f noise.

1. INTRODUCTION

Flicker noise or just 1/f noise is such that its power spectral density (P.S.D.) varies with frequency approximately in the form \( S(f) = \frac{k}{f} \). It also receives the name “pink” noise in contraposition to “white” noise because light with a spectral density of the same form would appears as a weak pink color to the human eye. 1/f noise is present in most natural phenomena. Wentai Li [13] reports an extensive bibliography grouped into several categories, ranging from 1/f noise in electronics, biology and chemical systems to 1/f noise in music and network traffic. However, it is in electronic devices that 1/f noise has received most attention and particularly in MOS transistors because it becomes an important limitation in circuit design, especially for instrumentation.

The physics behind flicker noise in the MOS transistor is still a topic of discussion. It is more or less accepted that the sources of low frequency noise are mainly carrier number and mobility fluctuations due to random trapping – detrapping of carriers in energy states near the surface of the semiconductor. However, the exact mechanism and the statistics of the resulting noise current, as well as how it is related to technology parameters like doping concentration or surface quality, are still not clear.

From the designer’s perspective it is desirable to have noise models that allow an accurate prediction of the noise power spectral density in terms of the polarization, size and technology of the transistor. These models should not be complex to compute and employ variables which are easily handled by a simulator, like currents or charges in the transistor nodes. Finally, these models should be simple because the adjustment of several technology parameters becomes difficult when limited noise measurements are available. It is also a condition for a good model to preserve the symmetry of the transistor, to be continuous in all the regions of operation and to give consistent results for the series – parallel association. A considerable effort has been made to set up MOS transistor models providing them with the above characteristics; but a consistent and continuous associated flicker noise model has not yet been developed. As far as we know, there is no reference in the literature to the series-parallel association properties of MOSFET’s noise models, and in fact most of the usual flicker noise models are not consistent in this sense.

In this paper we develop a physics based model for the flicker noise in MOS transistors. First we review noise modeling for the series-parallel association of devices, starting with resistors and ending with MOS transistors. After that we present a brief description of common noise models used for electrical simulation, discussing their series-parallel association properties as well as their behavior along the different (weak (W), moderate (M), strong (S)) inversion regions of operation of the transistor. Thirdly, we will discuss a simple physics based model of the low frequency noise. Based on a few hypothesis, we
will deduce an expression for the noise due to carrier trapping-detrapping in energy states inside the oxide. This noise model results in the particular 1/f dependence on the frequency of the P.S.D. The background physics is the same as that which has been presented in several papers on the subject [2-5], but with the aid of an advanced compact transistor model [8-10] we will be able to integrate exactly the noise contributions along the channel, resulting in a consistent, simple, and continuous 1/f noise model for the MOS transistor, valid in all the operation regions. Finally, we present some preliminary measurements of flicker noise in MOS transistors where we cover all the operation regions. We compare these experimental results with the theoretical expression here derived and finally present some conclusions.

2. SERIES-PARALLEL ASSOCIATION AND CONSISTENT NOISE MODELS.

The most fundamental electrical noise is the thermal noise present in every resistor due to the thermal random movement of charges. The spectral density is a white noise (does not vary with the frequency):

\[
S_i(f) = \frac{4kT}{R}, \quad (1)
\]

where we represent noise as a current source in parallel with the resistor. K is the Boltzmann’s constant, T the absolute temperature and R the resistor value. As shown in Fig.1, the noise formula (1) is consistent when applied to the series association of resistors. If we calculate the current variation \( \Delta i \) in Fig.1a due to currents \( i_{n_1} \) and \( i_{n_2} \) and then its spectral density \( S_{i_1} \), the result is the same as we would obtain applying Eq.(1) directly to the equivalent resistor \( R_{eq} \) (Fig.1b).

In Fig.2 a similar procedure for the case of parallel association is shown. In this case the equivalent noise source \( i_{n_{eq}} \) is the composition of the two non-correlated sources \( i_{n_1} \) and \( i_{n_2} \). The P.S.D. once again could have been calculated by applying Eq. (1) to the equivalent resistor \( R_{eq} \).

\[
\Delta i = \frac{R_{eq}}{R_{eq}} \frac{R_{i_1} + R_{i_2}}{R_{i_1} + R_{i_2}} \Rightarrow S_{\Delta i} = \frac{4kT}{R_{eq}} \quad (2)
\]

Fig.1: a) two resistors in series b) associated equivalent resistor and noise.

\[
i_{n_{eq}} = i_{n_1} + i_{n_2}
\]

\[
\Rightarrow S_{i_{n_{eq}}}(f) = 2kT \left( \frac{1}{R_{i_1}} + \frac{1}{R_{i_2}} \right) = \frac{2kT}{R_{eq}} \quad (3)
\]

Fig.2: a) two parallel resistors b) associated equivalent resistor.

Now consider the case of a MOS transistor. In Fig 3, a scheme where a single transistor is split into two parallel ones is shown. Small signal analysis is performed to calculate the equivalent noise current \( i_{n_{eq}} \). Considering non correlated noise sources \( i_1 \) and \( i_2 \), it follows:

\[
S_{i_{n_{eq}}}(f) = S_{i_{n_1}}(f) + S_{i_{n_2}}(f) \quad (4)
\]

A similar analysis may be performed for the series association of transistors (see Fig.4). In this case the small signal analysis leads to the conclusion that to be consistent, a model must verify:
\[ S_{i_{\text{eq}}}(f) = S_{i_{12}}(f) \left( \frac{1}{1+k} \right)^2 + S_{i_{11}}(f) \left( \frac{k}{1+k} \right)^2 \quad (5) \]

where \( S_{i_{11}}(f) \) and \( S_{i_{12}}(f) \) are the noise power spectral densities of the lower, the upper and the equivalent transistor respectively. The coefficient \( k \) is defined as

\[ k = \frac{g_{m_{s1}}}{g_{m_{D2}}} \]

In effect, considering the general expression for the source (drain) transconductance \( g_{m_{S(D)}} \) defined as the derivative of the drain current with respect to the source (drain) voltage [8]:

\[ g_{m_{S2}} = -\mu \frac{W}{L-d} Q_{d} \quad g_{m_{D1}} = -\mu \frac{W}{d} Q_{d}. \quad (6) \]

Eq. (5) can be written:

\[ S_{i_{\text{eq}}}(f) = S_{i_{21}}(f) \left( \frac{L-d}{L} \right)^2 + S_{i_{11}}(f) \left( \frac{d}{L} \right)^2 \quad (7) \]

As an example we will consider the classical model for thermal channel noise [1]:

\[ S_i = -\frac{4kT\mu Q_i}{L^2}, \quad (8) \]

where \( \mu \) is the effective mobility and \( Q_i \) is the total inversion charge in the channel. Substituting Eq. (8) into the series composition of thermal noises given in Eq. (7), we obtain:

\[ S_i = -\frac{4kT\mu Q_i}{L^2} \left[ \frac{Q_{i1}}{d^2} \left( \frac{L-d}{L} \right)^2 + \frac{Q_{i2}}{(L-d)^2} \left( \frac{L-d}{L} \right)^2 \right] = -\frac{4kT\mu Q_i}{L^2} \quad (9) \]

As expected, the classical thermal noise model of the MOSFET is consistent regarding the series (parallel) association.

### 3. COMMON SPICE MODELS FOR FLICKER NOISE.

Table I summarizes common noise models that are implemented in circuit simulators [11,12]. In the second and third column we state whether noise models are consistent with series and parallel association in the sense of Eq. (4,7). Note also that if this test is performed with a simulator not only the noise model should be consistent but also the DC and AC transistor models. In effect, noise analysis depends on DC bias calculation, small signal parameter values, as well as on the noise equivalent sources.
<table>
<thead>
<tr>
<th>MODEL</th>
<th>SERIES</th>
<th>PARALLEL</th>
<th>$S_{id}/I_o^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spice NLEV=0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{id} = K_f I_D^{AF} \frac{1}{C_{ox} \cdot L^2} \cdot \frac{1}{f}$</td>
<td>✓</td>
<td>Only if AF =1</td>
<td>× ( \text{Tends to 0 in W.I.} )</td>
</tr>
<tr>
<td>Spice NLEV=1:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{id} = K_f I_D^{AF} \frac{1}{C_{ox} \cdot W \cdot L} \cdot \frac{1}{f}$</td>
<td>✓</td>
<td>Only if AF =2</td>
<td>× ( \text{Remains constant in the whole operating range.} )</td>
</tr>
<tr>
<td>Spice NLEV=2,3:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{id} = K_f g_{m^2} \frac{1}{C_{ox} \cdot W \cdot L} \cdot \frac{1}{f_{EF}}$</td>
<td>×</td>
<td>Only under certain approximations.</td>
<td>✓ ( \propto g_{m^2}/W )</td>
</tr>
<tr>
<td>BSIM 3v3</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

See ref. [14] for description. (partial see note (1))

Note: According to BSIM 3v3 manual [14] it uses a different model for S.I. and W.I., both verify series and parallel association but this is not necessarily the case of their empirical composition used to model moderate inversion.

Table 1: The most common flicker noise models with series-parallel association properties and behavior along the operation regions of the ratio $S_{id}/I_o^2$.

In the third column we analyze the behavior of the ratio $S_{id}/I_o^2$ along the different operating regions of the transistor. This ratio is assumed to present a plateau for W.I. while it strongly decreases (proportional to $1/Q_i^2$) as the transistor goes deep in S.I. [4]. Note for example that Spice NLEV=0 predicts a perfect noise performance in W.I. with $S_{id}/I_o^2$ tending to 0. Spice NLEV=1 does not reflect the variations on the bias condition while Spice NLEV=2,3 in spite of presenting the plateau for W.I. does not present a decay in the $S_{id}/I_o^2$ ratio as strong as expected. The BSIM3v3 noise model [14] shows a correct behavior for the $S_{id}/I_o^2$ ratio from weak to strong inversion and is partially consistent for series and parallel association. However the BSIM3v3 noise model is not continuous [14] and it presents the additional difficulty of having 3 fitting parameters.

Modern designs tend to employ an increasing number of transistors in W.I., so an adequate model is required covering the whole range of bias conditions. In the following section we will present a model for flicker noise, based on the traditional number fluctuation theory but supported by an advanced compact transistor model [8-10], to arrive at a noise model that is consistent, continuous and simple.

4. A PHYSICS BASED MODEL FOR Flicker NOISE.

According to the classical number fluctuation theory, first we consider a small volume $dV = W . dx . dz$ inside the oxide (Fig.5). Traps in this small volume will capture and emit carriers in a random process. Consider only traps with energies within a narrow range between $E$ and $E + dE$, and let $\Delta N_v$ denote the fluctuations in the number of occupied traps in this narrow energy range and in the volume $dV$. The power spectral density of this fluctuation $S_{\Delta N_v}$ is given by [2]:

$$S_{\Delta N_v}(f) = N_v f_v (1 - f_v) \cdot \frac{4\tau}{1 + (2\pi f)^2} \cdot dV \cdot dE \quad (10)$$

where: $N_v(z,E)$ is the trap density (eV$^{-1}$·cm$^{-3}$).
\( f_i = \left(1 + e^{(E-E_{\text{Fermi}})/kT} \right)^{-1} \) is the trap occupancy function, \( E_{\text{Fermi}} \) is the electron quasi-Fermi level. \( \tau(z, E) \) is the trapping process time constant.

To calculate the P.S.D. of the total fluctuation \( \Delta N_v \) of the number of occupied traps in the volume of oxide of height \( t_{ox} \) over a small channel area \( dA = W \cdot dx \) we have to integrate Eq. (10) over the energy and the \( z \) coordinate:

\[
S_{\Delta N_v} (f) = \int_{E_c}^{E_{\text{ox}}} \int_{0}^{t_{ox}} N_i f_i (1 - f_i) \frac{4\pi}{1 + (2\pi f)^2 \tau^2} W dx dz dE 
\]

To evaluate the integral in Eq. (11) two hypothesis are made [3]:

- The oxide traps have a uniform spatial and energy distribution \( N_i(z, E) = N_i \).

  Because \( f_i (1 - f_i) \) is sharply peaked, it can be shown [3] that

\[
\int N_i f_i (1 - f_i) dE \approx N_i kT 
\]

- The time constant of the process is given by a tunneling process where \( \tau = \tau_0 e^{-\lambda \phi} \), \( \lambda = \hbar/\sqrt{8m^*\phi} \) is the tunneling constant for electrons, \( m^* \) their effective mass, \( \phi \) the energy barrier and \( \hbar \) is the Planck’s constant.

\[
\int_{0}^{t_{ox}} \frac{\tau}{1 + (2\pi f)^2 \tau^2} dz = \frac{\lambda}{\omega} (\tan^{-1}(\omega \tau(t_{ox})) - \tan^{-1}(\omega \tau(0)))
\]

(13a)

The time constant \( \tau \) has a very large dispersion, consequently [3]:

\[
\int_{0}^{t_{ox}} \frac{\tau}{1 + (2\pi f)^2 \tau^2} dz \approx \frac{\lambda}{4f}
\]

(13b)

Now Eq.(11) can be easily calculated:

\[
S_{\Delta N_v} (f) = kT \lambda N_i W dx, \frac{1}{f} = N_{\text{ox}} W dx, \frac{1}{f}
\]

(14)

where we have defined the number of effective traps \( N_{\text{ox}} = kT \lambda N_i \).

Eq.(14) predicts the P.S.D. of total occupied traps over a channel region \( dA \). These variations produce also small variations in the number \( N \) of carriers in the channel in the area \( dA \). The ratio between both quantities is denoted \( r = \frac{\delta N}{\delta N_v} \), and it has a general expression deduced by a simple charge balance [4]:

\[
r = \frac{C_i'}{C_i + C_{\alpha}'} + C_i'
\]

(15)

\( C_i \) is the inversion layer capacitance per unit area that varies from \( C_i = -\frac{Q_i}{\phi} \) in W.I. to \( C_i' = -\frac{Q_i}{2\phi} \) in S.I. [9]. \( C_{\alpha} \) is the depletion layer capacitance per unit area and \( C_{\alpha} \) is the oxide capacitance per unit area. The P.S.D. of the fluctuations in the number of carriers in the channel in the area \( dA \) is given by:

\[
S_\Delta N(f) = r^2 S_{\Delta N_v} (f)
\]

(16)

But along the \( x \) coordinate the \( r \) ratio and the sheet resistance of the channel may vary significantly and thus \( S_\Delta N(f) \). For example in strong inversion with the channel pinched off, the channel charge density at the drain \( Q_{id} \) is negligible in magnitude in comparison to the channel charge density at the source \( Q_{is} \). Traditional approaches to physics based models for MOS flicker noise try to integrate noise contribution of all \( dA \) elements along the channel but fail to do it with wide generality. What is usual is to examine different operation regions like strong inversion, weak inversion or linear region separately, and with the adequate approximation for each case perform the integration along the channel. To overcome this difficulty

Fig.5: MOS transistor scheme and a small volume \( dV = W \cdot dx \cdot dy \cdot dz \) into the oxide.
we will employ the ACM model for the transistor [8-10]. The fundamental approximation of this model is the linear dependence of the inversion charge density on the surface potential $\phi_S$, which encompasses the weak, moderate, and strong inversion regions

$$dQ_i = (C_{ds} + C_{as})d\phi_S = nC_{as}d\phi_S$$  \hspace{0.5cm} (17)

where $n$ is the slope factor, slightly dependent on the gate voltage. The drain current in a long-channel transistor is calculated with the aid of Eq.(17) and the charge-sheet approximation [1]:

$$I_D = \frac{\mu W}{nC_{as}}(-Q_i + \phi_i nC_{as}) \frac{dQ_i}{dx}$$  \hspace{0.5cm} (18)

Now let us return to the case of a noisy element in the transistor. We separate the channel into 3 series elements: the upper transistor, the lower transistor, and the noisy element modeled as a noisy resistor (Fig.6). If we use Eqs.(14) to (16) to calculate the noise current $\Delta i$ produced by the element $dA$, then it is easy to calculate by a simple small signal analysis the resulting drain current noise $\Delta I_D$:

$$\Delta I_D = \frac{\Delta x}{L} \Delta i$$  \hspace{0.5cm} (20)

Thus if we have the noise current P.S.D. $S_i(x, f)$ of the current $\Delta i$ of a small channel area $dA$, then the total noise current of the transistor is:

$$S_{II}(f) = \frac{1}{L} \int_0^L S_i(x, f)dx$$  \hspace{0.5cm} (21)

Equations (20),(21) have a wide generality and could be employed for any noise current model for the noise of single channel elements like Hooge’s model [7], empirical model [1], or thermal noise model of Eq.(8). Note also that the use of Eq.(21) to compute total channel noise, inherently result in a consistent noise model in the sense of series association.

$$\Delta i = I_D \cdot \frac{\Delta \sigma}{\sigma}$$  \hspace{0.5cm} (22)

where $\sigma$ denotes the conductivity of the channel element $dA$. This conductivity varies with the carrier density in the channel and also with the effective mobility $\mu_{eff}$ of carriers in the surface. For the sake of simplicity we will consider only the former factor but the analysis could be extended to include fluctuation in the mobility [5] with the
penalty of introducing two more constants to adjust. We prefer to have a single constant to be adjusted in the noise model.

$$\frac{\Delta \sigma}{\sigma} = \frac{\Delta N}{N} \Rightarrow \frac{S_{\Delta \sigma}}{\sigma^2} = \frac{S_{\Delta N}}{N^2}$$  \hspace{1cm} (23)$$

With (14) to (16), (22) and (23) we compute the equivalent noise current of the small channel section \( dA \). Then we substitute expression (17) and perform the integration of Eq.(21) to calculate the total noise current of the transistor due to the random capture-emission of carriers in the oxide traps:

$$S_{ID} = \frac{q^2 N_{ot} I_D^2}{W L} \cdot \frac{1}{f} \int_0^1 \frac{1}{n C_{ox} \phi_i - Q_i} \, dx =$$

$$= \frac{q^2 N_{ot} \mu I_D}{n C_{ox} L^2} \cdot \frac{1}{f} \int_{Q_{ID}}^{Q_{IS}} \frac{1}{n C_{ox} \phi_i - Q_i} \, dQ_i$$  \hspace{1cm} (24)$$

where the integration over the charge variable is carried out with the aid of Eq.(18). Now the noise current P.S.D. is calculated as:

$$\frac{S_{ID}}{I_D^2} = \frac{q^2 N_{ot} \mu}{L n C_{ox} I_D} \cdot ln \left[ \frac{n C_{ox} \phi_i - Q_{IS}}{n C_{ox} \phi_i - Q_{ID}} \right] \frac{1}{f}$$  \hspace{1cm} (25)$$

- In weak inversion, \( Q_{IS} \cdot Q_{ID} < n C_{ox} \phi_i \). Using the gate transconductance expression [8]:

$$g_m = \frac{\mu W}{nL} \left( Q_{ID} - Q_{IS} \right),$$  \hspace{1cm} (26)$$

and making a first order series expansion, it is possible to rewrite Eq.(25) as:

$$\frac{S_{ID}}{I_D^2} = \frac{q^2 N_{ot}}{W L n C_{ox}^2 \phi_i} \cdot \frac{g_m}{I_D} \cdot \frac{1}{f},$$  \hspace{1cm} (27)$$

where it is clear that the \( \frac{S_{ID}}{I_D^2} \) ratio has the same bias dependence as the \( gm/I_D \) term and consequently must present a plateau in the weak inversion region of operation.

- In strong inversion and in the linear region, 

  $$Q_{IS} \equiv Q_{ID} \equiv C_{ox} (V_g - V_T),$$

  and the first order expansion of the Eq.(25) leads to:

$$\frac{S_{ID}}{I_D^2} = \frac{q^2 N_{ot}}{W L C_{ox}^2 (V_g - V_T)^2} \cdot \frac{1}{f}$$  \hspace{1cm} (28)$$

5. MEASUREMENTS

In order to test the model of Eq.(25) some noise measurements have been performed for a saturated NMOS transistor from weak to strong inversion. The transistor, fabricated in a 2.4\( \mu \) technology, has an aspect ratio \( W/L=480\mu/16\mu \). We choose a wide transistor in order to be able to comfortably characterize weak inversion operation. The experimental setup is shown in Fig.8; the low noise amplifier is a Stanford Research SR560, the spectrum analyzer is a Hewlett Packard HP3582A, and different R values have been employed from 1k to 100k\( \Omega \). To measure the drain current and fix the node’s voltages we employed a Hewlett Packard HP4155 Semiconductor Parameter Analyzer. The gate voltage \( V_g \) is adjusted to obtain a desired drain current while the \( V_D \) value is adjusted to obtain 3.5Volts in the transistor drain that guarantees the saturation in all cases. The employment of the spectrum analyzer allows us to observe curves of the P.S.D. in terms of the frequency that in all cases have shown 1/f dependence as it was expected. However, \( S_{ID}(f) \) plots are not shown here and once the 1/f dependence was checked, we employed a narrow band analysis of the P.S.D. around an arbitrary low frequency to study the behavior of the flicker noise as the bias point changes. The selected frequency was 0.7Hz, an enough low frequency to neglect the effect of white noise. This procedure is the same employed by Reimbold [4] to observe the plateau in the \( S_{ID}/I_D^2 \) curve.

In Figure 9 the measured data as well as a comparison with the model of Eq.(25) are shown. The solid triangles represent the measured values \( S_{ID} \) of P.S.D. of the flicker noise for a frequency \( f=0.7Hz \). With these values we calculated the \( S_{ID}/I_D^2 \) ratio that is represented in the open triangles. To compare the measured values with the model of Eq.(25) we have to calculate the channel charge densities at source and drain. We measured the drain and source transconductances \( g_{mD}, g_{mS} \) employing the
In terms of the drain current. The ID-IDEG1 µ=1.4(A/V).

The dashed line in the plot of Fig.9 is the calculated S_ID/I_D² ratio from Eq.(25). In Fig.10 the transconductance gm and the \( \frac{gm}{I_D} \) ratio are shown. In Fig.11 the calculated channel charge densities \( Q_{ID}, Q_{IS} \) in terms of the drain current are shown.

Fig.8: Experimental setup for noise measurements.

Fig.9: Measured value \( S_O \) of the P.S.D. of the flicker noise at a frequency f=0.7Hz; measured and estimated ratio \( S_O/b \) in terms of the drain current b.

Fig.10: Transconductance gm and gm/b ratio for the test transistor in terms of Ic.

Fig.11: Measured channel charge density at the source and drain in terms of the drain current.

6. CONCLUSIONS

The problem of the consistent representation of the noise in series-parallel association of transistors has been presented as well as a discussion of the properties of common flicker noise models implemented in electric simulators. A flicker noise model, consistent and continuous in all the operation regions from weak to strong inversion has been developed. The new model is based in usual physical hypothesis but we use a procedure to integrate the contribution to the transistor noise current of all the noisy elements in the channel that inherently preserves the series association properties of the model. With the aid of an advanced compact transistor model, this integration procedure result in a simple, continuous, and consistent model for flicker noise in MOS transistors.

7. REFERENCES


