A CURRENT-BASED MODEL FOR THE MOS TRANSDISTOR
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ABSTRACT
This paper presents a physics-based model for the MOS transistor, suitable for circuit design and simulation and valid from weak to strong inversion. Each static or dynamic characteristic is accurately described by a single-piece function of two saturation currents.

1. INTRODUCTION
Analog IC designers search for MOSFET models in which simple single-piece expressions, with a minimal number of parameters, provide enough accuracy in all operating regions [1, 2]. The model should be charge-conserving [1] and preserve the intrinsic symmetry of the device [2]. In order to simplify hand calculations in circuit design and the procedure of parameters extraction, convenient key independent variables should be used in the model formulation. In this work, the model of refs.[3, 4], which satisfies all the above mentioned requirements, is entirely reformulated in terms of two saturation currents. Taking advantage of the transistor source-drain symmetry, the source and drain voltages and the current-to-transconductance ratio are also expressed as functions of two saturation currents.

2. FUNDAMENTALS
In [3, 4] we have presented a physics-based model for the MOSFET comprised of single-piece expressions for all the device characteristics. This model is valid in the whole inversion regime. The fundamental assumption of this model is the linear dependence of the inversion charge density \(Q'_s\) on the surface potential \(\phi_s\) [3-5], for a given gate-to-bulk voltage \(V_G\):

\[
dQ'_s = nC_{ox}'d\phi_s
\]

where \(C_{ox}'\) is the oxide capacitance per unit area and \(n\) is the slope factor, slightly dependent on the gate voltage.

Eqn.(1) has allowed the model in [3,4] to be fully formulated in terms of the inversion charge densities at the source \(Q'_s\) and drain \(Q'_d\) channel ends. Accordingly, the drain current \(I_D\) in a long channel device can be written as the difference between its forward \(I_F\) and reverse \(I_R\) saturation components:

\[
I_D = I_F - I_R \quad (2.a)
\]

\[
I_{F[R]} = \frac{W}{L} \frac{n\phi_s^2}{2} \left[ \left( \frac{Q'_s}{nC_{ox}'\phi_s} \right)^2 - \frac{2Q'_s}{nC_{ox}'\phi_s} \right] \quad (2.b)
\]

where \(\mu\) is the carrier mobility, \(\phi_s\) is the thermal voltage, \(W\) is the channel width and \(L\) is the channel length.

\[0\quad V_S\]

\[\rightarrow\]

\[I_R\quad I_D\quad I_F\quad V_D\]

\[\rightarrow\]

\[\text{forward saturation}\]

Fig. 1. Output characteristic of a long-channel NMOS transistor for constant \(V_S\) and \(V_D\). All the voltages are referred to the bulk terminal.

According to (2.b), the inversion charge density at the source (drain) has a one-to-one relationship to the drain current in forward (reverse) saturation. Therefore, the transistor characteristics in [3, 4] can be expressed in terms of the easily measurable variables \(I_F\) and \(I_R\). Such a feature is
extremely convenient for IC designers, since analog circuits are usually current-biased.

Fig.1 illustrates a typical output characteristic of a long-channel NMOS transistor in order to show the saturation components. In forward saturation, the drain current equals $I_d$, which is independent of the drain voltage $V_D$. Analogously, we verify that $I_D = -I_R$ in reverse saturation, with $I_R$ independent of the source voltage $V_S$.

3. MODEL FORMULATION

Eqn.(2.b) can be rewritten as:
\[ -\frac{Q_{(SS)}' \phi_t}{nC_{ox}} = \sqrt{1 + i_{f(t)}} - 1 \]
(3.a)

where
\[ i_{f(t)} = \frac{I_{(R)}}{I_S} \]
(3.b)
is the forward (reverse) normalized current [2] and
\[ I_S = \mu C_{ox} \frac{W}{L} n \frac{\phi_t^2}{2} \]
(3.c)
is the normalization current, which is four times smaller than the homonym presented in ref.[2].

In [2, 3], the source (drain) transconductance $g_{ms(d)}$ is given by:
\[ g_{ms(d)} = -\mu \frac{W}{L} Q_{SS}' \]
(4)
The substitution of (3.a) into (4) leads, after some algebra, to the universal relationship
\[ \frac{I_{(R)}}{\phi_t g_{ms(d)}} = \frac{1 + \sqrt{1 + i_{f(t)}}}{2} \]
(5)

which is independent of technology, gate voltage, dimension and temperature. Fig.2 illustrates the universality of this law. Experimental data obtained for long-channel MOS transistors from different technologies and biased by different gate voltages agree very well with the curve obtained using (5).

Since $g_{ms(d)}$, by definition, is the derivative of $-I_{(R)}$ with respect to $V_{SS'(d)}$, the relationship between source (drain) voltage and forward (reverse) normalized current is determined by integrating (5):
\[ \frac{V_F - V_{SS'(d)}}{\phi_t} = \sqrt{1 + i_{f(t)}} - \sqrt{1 + i_F} + \ln \left( \frac{1 + i_{f(t)}}{1 + i_F} - 1 \right) \]
(6)

In (6), $V_F$ is the “pinch-off” voltage, which can be defined as a quasi-linear function of $V_S$ [2, 3]. $i_F$ is the value of $i_f(t)$ corresponding to the pinch-off condition.

4. RESULTS

Table I presents the expressions of the drain current, total charges, small-signal parameters and applied voltages in terms of the forward and reverse normalized currents. These expressions result from applying (3.a) to the model of [3, 4]. Table I synthesizes the overall behavior of a large and long channel device from weak to strong inversion. It is remarkable that only three parameters ($I_S$, $C_{ox}$ and $n$) are enough to characterize the small-signal parameters of the MOSFET. Narrow and short channel effects can be modeled as in [2].

The accuracy of the model described in Table I is confirmed in Figs. 3 and 4, which show the measured and simulated values of the source transconductance and the drain current in saturation.

![Fig.2. Forward current to transconductance ratio vs. forward normalized current of NMOSFETs with: $t_{ox} = 55 \, \text{Å}; \, \square (V_G = 1.0 \, \text{V}) \, \text{and} \, \circ (V_G = 2.0 \, \text{V})$ $t_{ox} = 280 \, \text{Å}; \, * (V_G = 1.0 \, \text{V}) \, \text{and} \, \Delta (V_G = 2.0 \, \text{V})$ our model - expression (5).]

![Fig.3. Source transconductance of an NMOS transistor with $t_{ox} = 280 \, \text{Å} \, \text{and} \, W = L = 25 \, \mu\text{m} \ (V_G = 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 3.6, 4.2 \, \text{and} \, 4.8 \, \text{V}; \, V_D = V_G)$: (—) simulated curves; (o) measured curves.]

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Finally, Fig. (5) compares the transistor intrinsic (trans)capacitances simulated in SMASH according to our model and according to the BSIM model. On the contrary to the BSIM model, our model assuses the continuity and smoothness of these (trans)capacitances in the whole inversion regime.

5. CONCLUSIONS

We have presented a MOSFET model completely formulated in terms of the forward and the reverse normalized currents and very appropriate for analog circuit design and simulation. This model takes into account transistor symmetry and charge conservation. The model requires few parameters and its accuracy has been demonstrated by comparing simulated and measured characteristics in all regions of operation.

Fig. 5. Simulated intrinsic capacitances of a long-channel NMOS transistor with $t_{os} = 280 \text{ Å}$ and $W = L = 25 \mu m$ ($V_S = 0$ and $V_D = 2.0$ V): — our model - Table I; (- - -) BSIM model. ($C_{gs} = C_{gb} + C_{gs} + C_{gd}$).

ACKNOWLEDGMENTS. The authors would like to thank CAPES and CNPq (from the Brazilian Ministries of Education and Science and Technology) for the financial support, the LPCS, Grenoble, for supplying the test devices, Dolphin Integration Company, Grenoble, for licensing SMASH simulator, and engineer S.M. Acosta for device characterization.

REFERENCES


Table I - Expressions for MOSFET Static and Dynamic Characteristics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_D$</td>
<td>$I_S (i_i - i_r)$</td>
</tr>
<tr>
<td>$Q_I$</td>
<td>$-C_{ox} n \phi_i \left[ \frac{2}{3} \left( \sqrt{1+i_r} - \sqrt{1+i_i} \right) + \frac{1}{\sqrt{1+i_r} + \sqrt{1+i_i}} - 1 \right]$</td>
</tr>
<tr>
<td>$Q_B$</td>
<td>$-\frac{n-1}{n} Q_I - C_{ox} \left[ (n-1) \phi_i (\sqrt{1+i_p} - 1) + \frac{\gamma^2}{2(n-1)} \right]$</td>
</tr>
<tr>
<td>$Q_S$</td>
<td>$-C_{ox} n \phi_i \left[ \frac{2}{15} \left( \frac{3(1+i_r)^3 + 6(1+i_r) \sqrt{1+i_r} + 4(1+i_r)^2 (1+i_i) + 2(1+i_r)^3}{\sqrt{1+i_r} + \sqrt{1+i_i}} \right) \right] - \frac{1}{2}$</td>
</tr>
<tr>
<td>$Q_D$</td>
<td>$Q_I - Q_S$</td>
</tr>
<tr>
<td>$g_{m(h)}$</td>
<td>$\frac{2I_S}{\phi_i} \left( \sqrt{1+i_f} - 1 \right)$</td>
</tr>
<tr>
<td>$\overline{g_{mg}}$</td>
<td>$\frac{g_{ms} - g_{md}}{n}$</td>
</tr>
<tr>
<td>$C_{gs(h)}$</td>
<td>$\frac{C_{ox}}{3} \left[ 1 - \frac{1}{\sqrt{1+i_f}} \right] \left[ 1 - \frac{1}{\sqrt{1+i_f}} \left( \frac{1+i_i}{\sqrt{1+i_f} + \sqrt{1+i_i}} \right)^2 \right]$</td>
</tr>
<tr>
<td>$C_{gb} = C_{bg}$</td>
<td>$\frac{n-1}{n} \left( C_{ox} - C_{gw} - C_{gd} \right)$</td>
</tr>
<tr>
<td>$C_{hs(h)}$</td>
<td>$(n-1)C_{gs(h)}$</td>
</tr>
<tr>
<td>$C_{ss}$</td>
<td>$\frac{C_{ox}}{15} \left( \sqrt{1+i_f} - 1 \right)^3 \left( \sqrt{1+i_r} - 8(1+i_r) \right) \left( \sqrt{1+i_r} + \sqrt{1+i_i} \right)$</td>
</tr>
<tr>
<td>$C_{sd}$</td>
<td>$\frac{4}{15} \left( \sqrt{1+i_r} - 1 \right)^2 \left( \sqrt{1+i_r} - 1 \right)^3 \left( \frac{1+i_r}{\sqrt{1+i_f} + \sqrt{1+i_i}} \right)$</td>
</tr>
<tr>
<td>$C_{eg}$</td>
<td>$\frac{C_m - C_{ed}}{n}$</td>
</tr>
<tr>
<td>$C_{eb}$</td>
<td>$(n-1)C_{eg}$</td>
</tr>
<tr>
<td>$V_F - V_{S(D)}$</td>
<td>$\phi \left[ \frac{1}{\sqrt{1+i_f} - 1} + \ln \left( \frac{\sqrt{1+i_f} - 1}{\sqrt{1+i_p} - 1} \right) \right]$</td>
</tr>
</tbody>
</table>

$C_{ox} = WLC'_{ox}$