Charge-Based Formulation of Thermal Noise in Short-Channel MOS Transistors

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ABSTRACT

In this communication we develop a charge-based formulation for the thermal noise in short-channel transistors. We arrive at a closed expression for the channel thermal noise including the velocity saturation effect for all the operating regions of the MOSFET. Such a result is increasingly important not only in strong inversion but also in moderate and weak inversion owing to technology scaling into the deep submicron regime. We based the calculation of the thermal noise on the impedance field method [4], which allows calculating the effect of local fluctuations on the measurable noise at the device terminals. Our derivation follows the same procedure used in [2], but rather than constraining the result to strong inversion, our quite general result is expressed in terms of the inversion charge densities at the source and drain terminals.

Keywords: MOSFET, MOSFET model, velocity saturation effect, thermal noise

1 INTRODUCTION

In this introduction, we review the dc equations of the MOSFET as well as define some normalization quantities.

The Pao-Sah expression of the MOSFET drain current I_D is

$$I_D = -\mu W Q_I^{'} \frac{dV_C}{dy} \tag{1}$$

where μ is the mobility and W is the channel width. The relationship between inversion charge density $Q_I^{'}$ and channel potential V_C is given [1] by the unified charge control model (UCCM) as

$$V_P - V_C = \phi_t \left[\frac{Q_I'}{Q_{IP}'} - 1 + \ln\left(\frac{Q'_I}{Q_{IP}'} \right) \right] \tag{2}$$

where $Q_{IP}^{'} = -nC_{ox}^{'}\phi_t$ is the value of $Q_I^{'}$ at pinch-off. V_P is the channel potential at pinch-off, n is the slope factor ,and ϕ_t is the thermal voltage.

To account for the mobility dependence on the longitudinal field, we use the following expression

$$\mu = \frac{\mu_0}{1 - \frac{E}{E_{cr}}} = \frac{\mu_0}{1 - \frac{\mu_0 E}{v_{\lim}}} \tag{3}$$

Using the linear relationship between surface potential and inversion charge density

$$dQ_I' = nC_{ox}^{'} d\phi_S \tag{4}$$

as in [5], and (2) , (3) and (4) in (1) ,yields

$$I_D = \frac{-2I_S}{1 - \varepsilon \frac{dq_I^{'}}{d\xi}} \left[q_I^{'} + 1 \right] \frac{dq_I^{'}}{d\xi} \tag{5}$$

where

$$q_I^{'} = \frac{Q_I^{'}}{Q_{IP}^{'}}, I_S = \frac{\mu_0 C_{ox}^{'} n\phi_t^2}{2} \frac{W}{L}, \xi = \frac{y}{L}, \varepsilon = \frac{\mu_0 \phi_t}{L v_{\lim}} \tag{6}$$

are the normalized inversion charge density, the specific current for low-field mobility, the normalized distance to the source and the short-channel factor, respectively. The integration of (5) along the transistor channel, from source to drain results in

$$i_d = \frac{\left(q_{IS}^{'} + q_{ID}^{'} + 2 \right)}{1 + \varepsilon\left(q_{IS}^{'} - q_{ID}^{'} \right)} \left(q_{IS}^{'} - q_{ID}^{'} \right) \tag{7}$$

for the drain current normalized to the specific current. The normalized inversion charge densities $q_{IS}^{'}$ and $q_{ID}^{'}$ in (7) are calculated at source and drain, respectively. The drain current in (7) is normalized to I_S.

2 THE CURRENT DIVISION PRINCIPLE

In order to calculate the channel noise, we now proceed as in [2], with an infinitesimal noise current source divided into two current sources. The effect of each one on the drain current is calculated using the impedance field method [4]. The channel conductances G_1 and G_2 are determined from

$$G_{1(2)} = \frac{dI_{1(2)}}{dV_C} = I_S \frac{di_{1(2)}}{dV_C} \quad (8)$$

Since we know how the current depends on the inversion charge and also that the relationship between charge and channel potential is given by the Unified Charge Control Model (UCCM) expression, we can solve (8) by using expressions (5) and (2), yielding

$$G_{1(2)} = G_0 \frac{di_{1(2)}}{dq_I^{'}} \frac{dq_I^{'}}{dV_C/\phi_t} = -G_0 \frac{di_{1(2)}}{dq_I^{'}} \frac{q_I^{'}}{q_I^{'}+1} \quad (9)$$

where $G_0 = I_s/\phi_t$ is the normalization conductance.

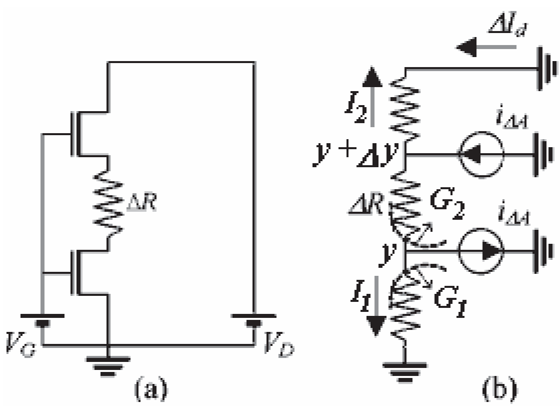

Figure 1: Transistor partition for calculating the contribution of elementary noise sources

The derivatives of the currents wrt the inversion charge density were calculated after we had computed the expressions for $i_{I(2)}$ by integrating expression (5) between the source (internal node y) and the internal node y (drain).

To obtain the effect of the elementary channel noise $i_{\Delta A}$ on the drain current ΔI_d, we superimpose the effects of the two elementary current sources as

$$\Delta I_d = \left(\left.\frac{G_2}{G_1+G_2}\right|_{\xi+\Delta\xi} - \left.\frac{G_2}{G_1+G_2}\right|_{\xi} \right) i_{\Delta A} \quad (10)$$

Note that the dc current through the two segments of the channel is the same; thus i_1=-i_2. The substitution of the derivatives of the currents into (9) leads to the following result for the current division

$$\frac{G_2}{G_1+G_2} = \frac{\xi + \varepsilon\left(q_{IS}^{'} - q_I^{'}\right)}{1+\varepsilon\left(q_{IS}^{'} - q_{ID}^{'}\right)} \quad (11)$$

The result in (11) is a generalization of the current division principle, derived under the assumption that the mobility follows the dependence on the longitudinal field as given by (3). For long channel devices, the current division simply states that, for a given current source injected into the channel, the fraction of the current that flows through the drain is proportional to the distance from the injection node to the source end of the channel. If, however, the saturation velocity phenomenon is significant, the current division principle should be modified according to expression (11), derived under the assumption of a mobility dependence on longitudinal field according to expression (3). At this point we apply (11) to a transistor operating in strong inversion, where $q_I^{'}$>>1. Thus, $q_I^{'} \cong \left(V_P - V_C\right)/\phi_t$, which leads to the following relationship for the current divider

$$\frac{G_2}{G_1+G_2} \cong \frac{y + \frac{V_{CS}}{E_{cr}}}{L + \frac{V_{DS}}{E_{cr}}} \quad (12)$$

where V_{CS} is the channel-to-source voltage. The result in (12) is equal to the one in [2].

Using expression (11) in (10) results in

$$\Delta I_d = \frac{\Delta\xi - \varepsilon\Delta q_I^{'}}{1+\varepsilon\left(q_{IS}^{'} - q_{ID}^{'}\right)} i_{\Delta A} = \frac{\Delta\xi\left(1-\varepsilon\frac{dq_I^{'}}{d\xi}\right)}{1+\varepsilon\left(q_{IS}^{'} - q_{ID}^{'}\right)} i_{\Delta A} \quad (13)$$

3 DRAIN CURRENT NOISE

In order to calculate the total drain current noise, we remind that the diffusion noise, i.e., the noise caused by the collisions of carriers with the lattice, is given [2] by

$$\overline{i_{\Delta A} i_{\Delta A}^{*}} = -4qDQ_I^{'}(y)\frac{W}{\Delta y}\Delta f \quad (14)$$

for an infinitesimally small thermal noise source localized between y and $y+\Delta y$. In (14), q is the electronic charge and D is the carrier diffusivity. In covalent semiconductors such as silicon, the diffusion constant decreases very slowly with increasing fields [6]. In our calculations, we will assume that the diffusivity is independent of the electrical field. The use of a constant diffusivity in (14) implicitly accounts for the hot carrier case in which the carriers are heated by the electrical field [2].

To normalize equation (14), the constant diffusivity can be expressed in terms of the low-field mobility, as given by Einstein relationship $D = D_0 = \mu_0\phi_t$. Rewriting (14) in normalized form, we then get

$$\frac{\overline{i_{\Delta A} i_{\Delta A}^{*}}}{4kT\Delta f} = \frac{2G_0}{\Delta \xi} q_I^{'}. \tag{15}$$

We can proceed now to determine the total noise current by adding the contribution of all channel elements, i.e.

$$I_d = \sum_{channel} \Delta I_d \tag{16}$$

In order to evaluate the mean square value of the drain current, we use (13) and (16) and assume that the elementary thermal noise sources are uncorrelated, which results in the following summation along the channel

$$\overline{I_d I_d^{*}} = \sum_{channel} \left(\frac{\Delta \xi \left(1 - \varepsilon \frac{dq_I^{'}}{d\xi} \right)}{1 + \varepsilon \left(q_{IS}^{'} - q_{ID}^{'} \right)} \right)^2 \overline{i_{\Delta A} i_{\Delta A}^{*}} \tag{17}$$

Substituting expression (15) into (17) and making $\Delta\xi \to 0$, the summation along the channel after applying (5) becomes

$$\frac{\overline{I_d I_d^{*}}}{4kTG_0\Delta f} = -\frac{4}{i_d} \frac{\int_{source}^{drain} q_I^{'} \left(q_I^{'} + 1 \right) \left(1 - \varepsilon \frac{dq_I^{'}}{d\xi} \right) dq_I^{'}}{\left[1 + \varepsilon \left(q_{IS}^{'} - q_{ID}^{'} \right) \right]^2} \tag{18}$$

The integral in (18) can be separated into two terms as in [2], the first one corresponding to thermal noise under equilibrium (Einstein relationship is assumed to hold for high electric fields) while the second one corresponds to noise added by the heating effect, i.e.,

$$\overline{I_d I_d^{*}} = \overline{I_d I_{d,equ}^{*}} + \overline{I_d I_{d,heat}^{*}} \tag{19}$$

$$\frac{\overline{I_d I_{d,equ}^{*}}}{4kTG_0\Delta f} = -\frac{4}{i_d} \frac{\int_{source}^{drain} q_I^{'} \left(q_I^{'} + 1 \right) dq_I^{'}}{\left[1 + \varepsilon \left(q_{IS}^{'} - q_{ID}^{'} \right) \right]^2} \tag{20}$$

$$\frac{\overline{I_d I_{d,heat}^{*}}}{4kTG_0\Delta f} = \frac{4\varepsilon}{i_d} \frac{\int_{source}^{drain} q_I^{'} \left(q_I^{'} + 1 \right) \frac{dq_I^{'}}{d\xi} dq_I^{'}}{\left[1 + \varepsilon \left(q_{IS}^{'} - q_{ID}^{'} \right) \right]^2} \tag{21}$$

If η is defined as

$$\eta = \frac{q_{ID}^{'} + 1}{q_{IS}^{'} + 1}, \tag{22}$$

the integration of (20) along the channel gives

$$\frac{\overline{I_d I_{d,equ}^{*}}}{4kTG_0\Delta f} = 2 \frac{\left[\frac{2}{3} \frac{\left(q_{IS}^{'} + 1 \right)\left(1 + \eta + \eta^2 \right)}{\left(1 + \eta \right)} - 1 \right]}{1 + \varepsilon \left(q_{IS}^{'} + 1 \right)\left(1 - \eta \right)} \tag{23}$$

One should note that the term in square brackets is the normalized inversion charge for a long-channel transistor.

Calculating $dq_I^{'}/d\xi$ from (5) and substituting the result into (21) we find that the integration along the channel leads to

$$\frac{\overline{I_d I_{d,heat}^{*}}}{4kTG_0\Delta f} = \frac{\varepsilon}{\left[1 + \varepsilon \left(q_{IS}^{'} - q_{ID}^{'} \right) \right]^2} \left\{ q_{IS}^{'\,2} - q_{ID}^{'\,2} + \varepsilon i_d \left(q_{IS}^{'} - q_{ID}^{'} \right) - \varepsilon i_d \left(1 - \frac{\varepsilon i_d}{2} \right) \ln \left(\frac{q_{IS}^{'} + 1 - \frac{\varepsilon i_d}{2}}{q_{ID}^{'} + 1 - \frac{\varepsilon i_d}{2}} \right) \right\} \tag{24}$$

In the linear region, for low drain-source voltages, the inversion charge along the channel is almost constant and, therefore, the transistor noise is similar to that of a long-channel device.

In the derivation that follows, we will calculate the so-called "white noise gamma factor" γ_{sat} for a saturated MOS transistor, defined by equation

$$\frac{\overline{I_d I_d^{*}}}{4kT\Delta f} = \gamma_{sat} g_{d0} \tag{25}$$

where g_{d0} is the MOSFET output conductance at $V_{DS}=0$.

In Fig. 2 the total transistor charge is shown, while Fig. 3 displays the dependence of γ_{sat} on both the inversion level at source and the short-channel factor. It is worth noting that, except for long-channel devices and not very high inversion levels, the usual expression for the channel noise based on the total charge does not hold.

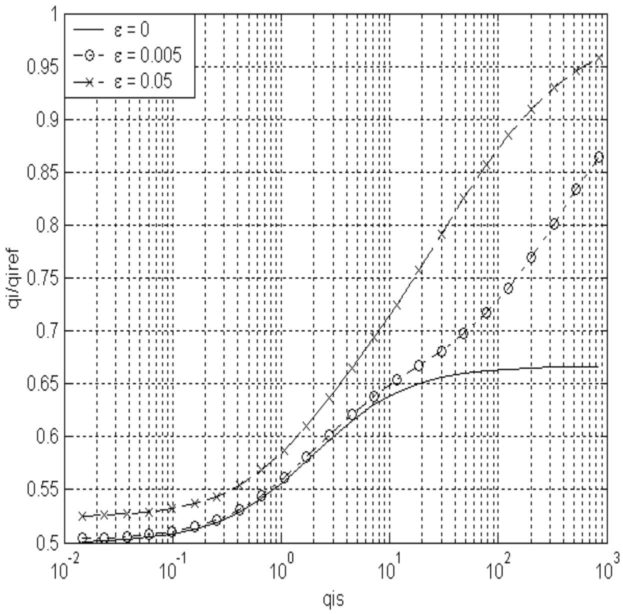

Figure 2 : Channel inversion charge (normalized to the inversion charge for $V_{DS} = 0$) vs. inversion charge density at source for short-channel coefficients equal to 0 (L→∞), 0.005 (L≈1μm) and 0.05 (L≈0.1μm).

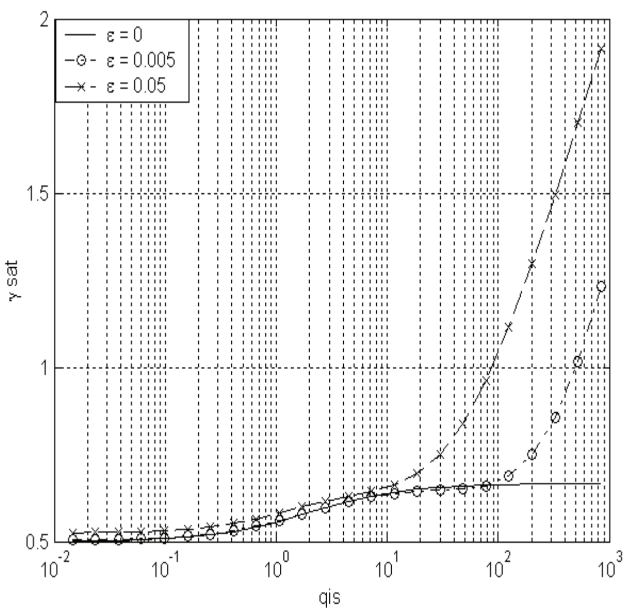

Figure 3 : Gamma factor vs. inversion charge density at source for short-channel coefficients equal to 0 (L→∞), 0.005 (L≈1μm) and 0.05 (L≈0.1μm).

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APPENDIX

The maximum current that can flow in the channel is limited by the maximum carrier velocity [3]. When electrons at the drain end of the channel reach the saturation velocity, the drain current is expressed as

$$I_D = -Wv_{\lim} Q'_{IDsat} \tag{26}$$

or, in normalized form

$$i_d = \frac{2}{\varepsilon} q'_{IDsat} \tag{27}$$

In (26), Q'_{IDsat} is the inversion charge density at the drain end of the channel. Equating (27) to (7) [3] allows one to calculate the inversion charge that corresponds to the onset of saturation, yielding

$$q'_{IDsat} = q'_{IS} - \frac{\varepsilon+1}{\varepsilon}\left(\sqrt{1+\frac{2\varepsilon}{(\varepsilon+1)^2}q'_{IS}} - 1\right) \tag{28}$$

The value of q'_{IDsat} given above is required to avoid that the calculation of the inversion charge at the drain end falls off below the saturation charge.