Interrelations between Threshold Voltage Definitions and Extraction Methods

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ABSTRACT

This paper presents a brief discussion on the main MOSFET definitions of threshold voltage available in the literature and associated extraction methodologies. We have taken advantage of the Advanced Compact MOSFET (ACM) model, which accurately relates surface potential ϕ_S to inversion charge density Q'_I in all regions of operation. A new robust and precise extraction method based on the transconductance-to-current ratio characteristic is reviewed, compared with already existing methods, and experimentally verified in a 0.18 µm CMOS technology.

Keywords: parameter extraction, threshold voltage, MOSFET model

1 INTRODUCTION

The threshold voltage V_T is a fundamental parameter in the modeling and characterization of MOS transistors. V_T represents a physical change in the current flow through the device as it goes from weak to strong inversion operation modes. Since this transition is very gradual, no critical point can be directly identified in the I_D vs. V_G characteristic as the onset of strong inversion. Consequently, different definitions of threshold voltage have been presented in the literature [1].

To analyze a V_T extraction procedure it is essential to use a model that includes both the drift and diffusion transport mechanisms, because both phenomena are important near the threshold condition. To shed some light on the V_T-extraction problem we will use a one-equation-all-regions model [2, 3] to calculate the band bending, total inversion charge at threshold, and the slight differences among the threshold voltages for the main extraction procedures.

In this paper we give a summary of a method of V_T determination in the linear region (low drain-to-source voltages) based on the g_m/I_D characteristic [4]. This new methodology is compared, regarding definitions and experimental results, with the traditional ELR (Extrapolation in the Linear Region), the TC/SDL (Transconductance Change / Second Derivative Logarithmic) and the constant-current (CC) methods for threshold voltage extraction, also in the linear region.

2 THE ACM MODEL

The ACM model consists of simple, accurate, and single equations that represent the device behavior in all regimes of operation, using well-known physical parameters [2, 3]. The ACM model is strongly based on two physical features of the MOSFET structure: the charge sheet model and the incrementally linear relationship between the inversion charge density Q'_I and the surface potential ϕ_S [2, 3]:

$$Q'_I \cong C'_{ox} n(\phi_S - \phi_{Sa}) \tag{1a}$$

$$n = 1 + \frac{C'_b}{C'_{ox}} = 1 + \frac{\gamma}{2\sqrt{\phi_{Sa} - \phi_t}} \tag{1b}$$

$$\phi_{Sa} - \phi_t = \left(\sqrt{V_G - V_{FB} - \phi_t + \frac{\gamma^2}{4}} - \frac{\gamma}{2}\right)^2 \tag{1c}$$

In (1), C'_{ox} and C'_b are the oxide and depletion capacitance per unit area, respectively, n is the slope factor, slightly dependent on the gate voltage V_G, γ is the body factor, ϕ_t is the thermal voltage and V_{FB} is the flat-band voltage. ϕ_{sa}, given by (1c), is the value of the surface potential deep in weak inversion, neglecting the inversion charge. Unless stated otherwise, the voltages herein are referred to the substrate.

The channel charge density for which the diffusion current equals the drift current is designated the **pinch-off charge density** Q'_{IP} [2, 3]:

$$Q'_{IP} = -nC'_{ox}\phi_t \tag{2}$$

The channel-to-substrate voltage (V_C) for which the channel charge density equals Q'_{IP} is called the **pinch-off voltage** V_P [2]:

$$V_P = \phi_{Sa} - \phi_0 \tag{3a}$$

$$\phi_0 = 2\phi_F + \phi_t\left[1 + ln\left(\frac{n}{n-1}\right)\right] \tag{3b}$$

In (3b), ϕ_F is the Fermi potential of the substrate.

Variable	Charge-Based Expression	Current-Based Expression	
I_D	$I_S\left(q'_{IS}-q'_{ID}\right)\left(q'_{IS}+q'_{ID}+2\right)$	$I_S(i_f\text{-}i_r)$	(4)
g_m	$\frac{2I_S}{n\phi_t}\left(q'_{IS}-q'_{ID}\right)$	$\frac{2I_S}{n\phi_t}\left(\sqrt{1+i_f}-\sqrt{1+i_r}\right)$	(5)
$V_P\text{-}V_{S(D)}$	$\phi_t\left[q'_{IS(D)}-1+ln\left(q'_{IS(D)}\right)\right]$	$\phi_t\left[\sqrt{1+i_{f(r)}}-2+ln\left(\sqrt{1+i_{f(r)}}-1\right)\right]$	(6)

Table 1: Expressions of the ACM model.

The expressions of ACM model to be used in this work are summarized in Table 1, where $V_{S(D)}$ is the source(drain)-bulk voltage, I_D is the drain current, $g_m = \partial I_D / \partial V_G$ is the gate transconductance, q'_{IS} and q'_{ID} represent the charge densities normalized with respect to Q'_{IP} and i_f and i_r are the forward and reverse saturation components of the current normalized with respect to the specific current I_S, given by:

$$I_S = \mu C'_{ox} n \frac{\phi^2}{2} \frac{W}{L} \qquad (7)$$

In (7) μ is the effective mobility, W is the effective channel width and L is the effective channel length. The expression of the surface potential ϕ_S can be derived from (1a), (3) and (6), resulting in

$$\phi_S = 2\phi_F + V_C + \phi_t\, ln\left(\frac{n}{n-1} q'_I\right) \qquad (8)$$

where q'_I is the normalized inversion charge density.

3 THRESHOLD VOLTAGE DEFINITIONS AND ASSOCIATED EXTRACTION PROCEDURES

3.1 Classical definition of the threshold voltage

V_{T0} is the gate voltage for which the electron concentration at the semiconductor interface equals the hole concentration in the bulk or, equivalently, $\phi_S = 2\phi_F$. Using this value for the surface potential in (8), one finds that the normalized charge $q'_{IT0} = (n-1)/n$ for $V_G = V_{T0}$ and $V_C = 0$. Substituting q'_{IT0} for q'_{IS} in (4) and (5) results in

$$\frac{g_m}{I_D} / \left(\frac{g_m}{I_D}\right)_{\max} = \frac{n}{2n-1} \qquad (9)$$

Expression (9) means that the determination of the classical threshold voltage from the relative (to the maximum) transconductance-to-current ratio requires the accurate determination of the slope factor n for values of gate voltage around the threshold voltage.

3.2 Threshold definition by extrapolation of strong inversion current characteristic

The definition on which the ELR method is founded is not clearly stated and simply arises from the fitting of measured drain current to an asymptotic strong inversion approximation. The ELR method assumes that, for low values of V_{DS} and in strong inversion ($i_f >> 1$), $I_D \propto (V_G - V_{Text})$ where V_{Text} is the extrapolated threshold voltage. The definition of V_{Text} along with expressions (4) and (5) and assuming $q'_{IS} \cong q'_{ID}$ (for $V_{DS} << \phi_t$) yields

$$V_G - V_{Text} = \frac{I_D}{g_m} = n\phi_t(q'_{IS} + 1) \qquad (10a)$$

Now, using (6) with $V_S = 0$ and the definitions of V_P in (3a), ϕ_{Sa} in (1c) and n in (1b) we obtain

$$V_{Text} = V_{FB} + (n-1)\phi_{Sa} + n\left[\phi_0 + \phi_t\left(ln\, q'_{IS} - 2\right)\right] \qquad (10b)$$

Even though the ELR method is very simple, it is prone to the influence of some factors neglected in the above analysis, such as mobility degradation due to transversal field, series resistances of source and drain, and the nonlinear relationship between inversion charge density and gate voltage [5].

3.3 Threshold definition by maximum of $\partial g_m / \partial V_G$ or minimum of $\partial^2 \ln I_D / \partial V_G^2$

A conceptually correct method to determine the (approximate) threshold voltage is based on the transconductance change (TC) [6] and consists of measuring the variation in g_m with respect to V_G and determining the maximum of this variation. From (6) and for $V_{DS} << \phi_t$:

$$\frac{V_{DS}}{\phi_t} = \left(q'_{IS} - q'_{ID}\right)\frac{q'_{IS}+1}{q'_{IS}} \qquad (11a)$$

Substituting (5) into (11a) we obtain

$$g_m = \mu C'_{ox} \frac{W}{L} V_{DS} \frac{q'_{IS}}{q'_{IS}+1} \qquad (11b)$$

Assuming the mobility to be constant, using (6) and substituting $dV_P/dV_G = 1/n$, we find that:

$$\frac{dg_m}{dV_G} = \mu C'_{ox}\frac{W}{L}\frac{V_{DS}}{n\phi_t}\frac{q'_{IS}}{\left(q'_{IS}+1\right)^3} \qquad (11c)$$

The derivative of the transconductance is maximum for $q'_{IS} = 0.5$ or, equivalently, $Q'_{IS} = 0.5Q'_{IP}$. Assuming that the variation of n with the gate voltage is negligible, the expression that relates the SDL and TC methods for small V_{DS} is:

$$\frac{dg_m}{dV_G} = -2I_S\frac{V_{DS}}{\phi_t}\frac{d^2 \ln I_D}{dV_G^2} \qquad (12)$$

From (12), we can conclude that, for low values of V_{DS}, the threshold voltages determined by the TC and SDL methods are quite close to each other. One major drawback of these methods is the need to calculate the usually extremely noisy second order derivative of the current.

3.4 Threshold voltage definition by the constant current method

In the CC method, the gate voltage, at which the drain current normalized by the transistor aspect ratio (W/L) equals a given value $I_{D.CC}$, is defined as the threshold voltage. A choice of $I_{D.CC}$ based on the nominal values of mobility and gate oxide capacitance results in a value of the threshold voltage very close to the classical definition. The substitution of the value of $\left(q'_{IS} - q'_{ID}\right)$ given by (4) into expression (11a) results in

$$I_D = 2I_S\frac{V_{DS}}{\phi_t}q'_{IS} = 2I_{SQ}\frac{W}{L}\frac{V_{DS}}{\phi_t}q'_{IS} \qquad (13)$$

where $I_{SQ} = I_S/(W/L)$ is the sheet specific current [1]. Once I_{SQ} is known, the threshold voltage can be chosen as the gate voltage at which, e.g., $q'_{IS} = 1$ or, equivalently, $I_D/(W/L) = 2I_{SQ}V_{DS}/\phi_t$. Except for a possible difficulty in determining the effective channel length and width, the CC method is quite attractive for its simplicity and accuracy.

4 EXTRACTION METHOD OF THE THRESHOLD VOLTAGE BASED ON THE g_m/I_D CHARACTERISTIC

The current-based expressions in (5) and (6) give

$$\frac{g_m}{I_D}\Bigg/\left(\frac{g_m}{I_D}\right)_{max} = \frac{2}{q'_{IS}+q'_{ID}+2} = \frac{2}{\left(\sqrt{1+i_f}+\sqrt{1+i_r}\right)} \qquad (14)$$

where $\left(g_m/I_D\right)_{max} = 1/\left(n\phi_t\right)$ is the value of the transconductance-to-current ratio deep in weak inversion.

In ACM model the threshold voltage is defined as the value of V_G for which the drift and diffusion components of the drain current are equal $\left(q'_I = 1\right)$. Applying this criterion to (14) for small V_{DS} ($q'_{IS} \cong q'_{ID}$ and $i_f \cong i_r$) and assuming n to be almost constant, allows extracting the threshold voltage from the g_m/I_D characteristic (Fig.1) by simply measuring the peak value of g_m/I_D and determining the gate voltage at which the value of g_m/I_D drops to one-half of the peak value. The slight variations of the slope factor and mobility with gate voltage are negligible over the required measurement range.

In order to account for the non-negligible value of $V_{DS,}$ q'_{ID} should be numerically evaluated through (6) for $q'_{IS} = 1$. The ratio $(g_{mg}/I_D)/(g_{mg}/I_D)_{max}$ is thus calculated using (14) and this value of q'_{ID}. For our measurements, we have chosen $V_{DS} = \phi_t/2$ which results in $q'_{ID} = 0.766$ and $g_m/I_D=0.5310(g_m/I_D)_{max}$ (circle in Fig.1) for $q'_{IS} = 1$. Since $V_S = 0$, the corrresponding value of V_G is the equilibrium threshold voltage V^*_{T0} according to the ACM model.

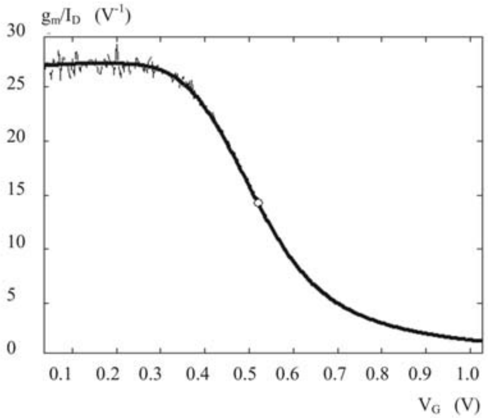

Figure 1: Transconductance-to-current ratio for $V_{DS} = 13mV$ and $V_{SB} = 0$. Dotted line: measured g_m/I_D; solid line: filtered g_m/I_D; circle: $g_m/I_D=0.5310(g_m/I_D)_{max}$. $L_m = W_m/100 = 0.2\ \mu m$ (mask channel length and width). TSMC - 0.18 μm technology

5 EXPERIMENTAL RESULTS

Measurements of the common-source characteristic in the linear region, with $V_S = 0$ and $V_{DS} = 13\ mV$ were taken for NMOS transistors of a 0.18 μm CMOS technology (TSMC) for several mask channel lengths L_m. In order to reduce the relative noise level and mismatching, each transistor is composed of the parallel association of ten devices. Table 2 exhibits the value of threshold voltage extracted for each test device through the g_m/I_D-based methodology, the ELR [1], the SDL [1], and the CC [1] methods.

L_m (μm)	V_{T0} (mV) - NMOSFET			
	ELR	SDL	g_m/I_D	CC
0.2	481	490	520	501
0.3	483	478	510	508
0.4	482	468	503	509
0.5	476	463	495	504
0.6	473	455	493	501
0.8	462	448	483	491
2.0	435	423	458	466

Table 2: Experimental results from g_m/I_D methodology, ELR, SDL, and CC methods for extracting the threshold voltage for TSMC 0.18 μm CMOS technology. Sheet specific current: $I_{SQN} = 168\ nA$.

6 SUMMARY AND CONCLUSIONS

The interrelations between the main threshold voltage definitions and extraction procedures have been clarified using a one-equation-all-regions MOSFET model, as summarized in Table 3. Unambiguous definitions of threshold have been emphasized and relative advantages/disadvantages of some common extraction procedures have been commented on.

A recent g_m/I_D-based methodology that provides a quick and reliable determination of the threshold voltage has been summarized. The new procedure determines the threshold voltage with negligible influence of parasitic resistances, short-channel effects and transversal field degradation, owing to the operation regime, linear region in weak and moderate inversion. The threshold voltage is evaluated according to a clear physical definition and its value closely agrees with the threshold voltage extracted through the ELR, SDL, and CC methodologies.

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Threshold Definition	**Physical Meaning**	**Value of ϕ_S at threshold**	**Value of Q'_I at threshold**	**Difference in V_{T0} relative to classical definition**
$\phi_S = 2\phi_F + V_C$	Surface concentration of electrons = bulk concentration of holes	$2\phi_F + V_C$	$-(n-1)C'_{ox}\phi_t$	0
$Q'_I = -nC'_{ox}\phi_t$	50% drop (relative to the peak) in the g_m/I_D curve	$2\phi_F + V_C + \phi_t \ln\left(\frac{n}{n-1}\right)$	$-nC'_{ox}\phi_t$	$\phi_t\left[1 + n\ln\left(\frac{n}{n-1}\right)\right]$
Extrapolated drain current	No clear physical meaning	Dependent on operating point	Not well defined	Dependent on operating point
$max\left(\frac{dg_m}{\partial V_G}\right)$	Peak on the second derivative curve	$2\phi_F + V_C + \phi_t \ln\left[\frac{n}{2(n-1)}\right]$	$-nC'_{ox}\frac{\phi_t}{2}$	$\phi_t\left\{1 + n\left[\ln\left(\frac{n}{2(n-1)}\right) - 0.5\right]\right\}$
Constant current	Equal drift and diffusion components of I_D	$2\phi_F + V_C + \phi_t \ln\left(\frac{n}{n-1}\right)$	$-nC'_{ox}\phi_t$	$\phi_t\left[1 + n\ln\left(\frac{n}{n-1}\right)\right]$

Table 3: Threshold definitions and associated meanings and features