

Analysis, Design and Simulation of analog/RF & Digital Circuits with Open-Source EDA Tools and Process Design Kit



https://github.com/Deni-Alves/MOSFET_model



Carlos Galup-Montoro Deni Germano Alves Neto Márcio Cherem Schneider Sylvain Bourdel



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'Scan me'





Part 1 Advanced Compact MOSFET Model: ACM2



Carlos Galup-Montoro

https://github.com/ACMmodel/MOSFET_model

"Scan me"

Outline

- Introduction: ACM timeline
- ACM2 model
- Long-channel: I_D and g_m/I_D models
- Long-channel: MOS basic amplifier design

ACM timeline

- 1993 ϕs -based model (SBMICRO, Campinas, Br)
- 1995 Long-channel charge-based model (SSE, Nov.)
- 1996 gm/ID model (SBMICRO, Aguas de Lindoia, Br)
- 1997 Unified current control model (ISCAS, Hong Kong)
- 1998 Most referenced ACM paper (JSSC, Oct.)
- 2000 ACM model in SMASH simulator (CICC, Orlando)
- 2021 4-parameter single-piece model (NorCAS, Oslo)
- 2023 ACM2 in VERILOG-AMS (NEWCAS, Edinburgh)



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The capacitive model of the MOSFET





- C_{ox} oxide capacitance per unit area C_{b} depletion capacitance per unit area
- **Q**₁ carrier charge density
- V_{T0} threshold voltage



$$\frac{\Delta \phi_s}{\Delta V_G} = \frac{C_{ox}}{C_{ox} + C_b} = \frac{1}{n}$$

$$\phi_s = 2\phi_F + \frac{V_G - V_{T0}}{n} = 2\phi_F + V_P$$

Drain current model: main simplifications

First approximation ACM2

 $dQ_I = nC_{ox}d\phi_s$

Second approximation ACM2



$$I_D = \mu W \left(-Q_I \frac{d\phi_s}{dy} + \phi_t \frac{dQ_I}{dy} \right)$$

drift

Diffusion

- **Q**₁ carrier charge density
 - W transistor width
 - μ carrier mobility
 - ϕ_t thermal voltage 26 mV @ 300K



Allows analytical integration for I_D

ACM2 current law

From the 3 approximations: normalized current vs. normalized charge densities at source and drain

$$\implies i_D = \frac{(q_S + q_D + 2)}{1 + \zeta(q_S - q_D)} (q_S - q_D)$$

normalization (specific) current

 $-nC_{ox}\phi_t$ thermal charge

$$i_D = I_D / I_S$$
 $I_S = \frac{W}{L} \mu_s n C_{ox} \frac{\phi_t^2}{2}$

$$q_{S(D)} = Q_{S(D)} / (-nC_{ox}\phi_t)$$

Short-channel parameter ζ :

$$\zeta = \frac{(\mu_s \phi_t / L)}{v_{lim}}$$

ratio of diffusion-related velocity to saturation velocity

Physics-based saturation



Saturation current due to saturation velocity of the carriers

$$I_{Dsat} = -WQ_{Dsat} v_{lim}$$

 Q_{Dsat} is the saturation inversion charge per unit area

or, using normalized variables

$$i_{Dsat} = \frac{2}{\zeta} q_{dsat}$$

"Carrier velocity approaches v_{sat} , but never reaches v_{sat} " Y.Taur TED March 2019

Physics-based saturation: design model

$$i_{Dsat} = \frac{2}{\zeta} q_{dsat} \qquad i_{Dsat} = \frac{(q_S + q_{Dsat} + 2)}{1 + \zeta(q_S - q_{Dsat})} (q_S - q_{Dsat})$$

$$q_{Dsat} = q_s + 1 + \frac{1}{\zeta} - \sqrt{\left(1 + \frac{1}{\zeta}\right)^2 + \frac{2q_S}{\zeta}}$$

or equivalently

$$q_s = \sqrt{1 + \frac{2}{\zeta}q_{dsat}} - 1 + q_{dsat}$$

The Unified Charge Control Model (UCCM)

$$\frac{V_P - V_C}{\phi_t} = q_I - 1 + \ln q_I$$
$$V_S \le V_C \le V_D$$
$$q_S \le q_I \le q_D$$

$$\frac{V_{DS}}{\phi_t} = q_S - q_D + \ln \frac{q_S}{q_D}$$

The "Regional" Weak (WI) and Strong Inversion (SI) Approximations WI

$$q_I \ll 1 \rightarrow V_P - V_C \ll \phi_t$$

$$q_I \cong e^{\frac{V_P - V_C + \phi_t}{\phi_t}}$$

 $q_I \gg 1 \rightarrow V_P - V_C \gg \phi_t$

$$q_I \cong \frac{V_P - V_C + \phi_t}{\phi_t}$$

Error <10% for $q_I < 0.22$

Error <10% for $q_I > 20$

UCCM including the effect of velocity saturation



Output curves including DIBL and v_{sat}



DIBL model: $V_T = V_{T0} - \sigma(V_S + V_D)$

Transisto r	W/L (μm/μm)	V_{T0} (mV)	$I_{S}(\mu A)$	n	σ	ζ
NMOS2V	5/0.18	528	5.52	1.37	0.025	0.056



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Long-channel charge-based model

For short channel MOS
$$I_D = I_S \frac{(q_S + q_D + 2)}{1 + \zeta(q_S - q_D)} (q_S - q_D)$$

$$\frac{V_P - V_S}{\phi_t} = q_S - 1 + \ln q_S \qquad \frac{V_{DS}}{\phi_t} = q_S - q_D + \ln \frac{q_S - q_{Dsat}}{q_D - q_{Dsat}}$$

For long channel MOS $\zeta = 0 \rightarrow q_{Dsat} = 0$ $I_D = I_S(q_S + q_D + 2)(q_S - q_D) = I_S[(q_S^2 + 2q_S) - (q_D^2 + 2q_D)]$

$$\frac{V_P - V_{S(D)}}{\phi_t} = q_{S(D)} - 1 + \ln q_{S(D)}$$

Obs: $\frac{dq_{S(D)}}{dV_G} = -\frac{1}{n} \frac{dq_{S(D)}}{dV_{S(D)}} \qquad \frac{dq_D}{dV_D} = -\frac{1}{\phi_t} \frac{q_D}{q_D + 1} \qquad \frac{dI_D}{dq_D} = -2I_S(q_D + 1)$ Bento Gonçalves Feb 2025

Small-signal transconductances

$$g_{md} = \frac{dI_D}{dV_D} = \frac{dI_D}{dq_D} \frac{dq_D}{dV_D} = 2I_S(q_D + 1) \frac{1}{\phi_t} \frac{q_D}{q_D + 1} = \frac{2I_S}{\phi_t} q_D$$
Symmetry
$$g_{ms} = \frac{2I_S}{\phi_t} q_S$$

$$g_m = \frac{dI_D}{dV_G} = \frac{dI_D}{dq_S} \frac{dq_S}{dV_G} + \frac{dI_D}{dq_D} \frac{dq_D}{dV_G}$$

$$g_m = \frac{g_{ms} - g_{md}}{n}$$

Unified Current Control Model (UICM)-I

$$I_D = I_S[(q_S^2 + 2q_S) - (q_D^2 + 2q_D)]$$
 (A)

(A) can also be written as

$$I_D = I_F - I_R = I_S [i_f - i_r]$$
^(B)

$$I_F$$
, I_R : forward and reverse currents

Unified Current Control Model (UICM)-II

$$\frac{V_P - V_{S(D)}}{\phi_t} = q_{S(D)} - 1 + \ln q_{S(D)}$$

$$q_{S(D)} = \sqrt{1 + i_{f(r)}} - 1$$
Normalized UICM
$$\frac{V_P - V_{S(D)}}{\phi_t} = \sqrt{1 + i_{f(r)}} - 2 + \ln\left(\sqrt{1 + i_{f(r)}} - 1\right)$$

$$\frac{V_{DS}}{\phi_t} = q_S - q_D + \ln \frac{q_S}{q_D} = \sqrt{1 + i_f} - \sqrt{1 + i_r} + \ln \left(\frac{\sqrt{1 + i_f} - 1}{\sqrt{1 + i_r} - 1}\right)$$

Bento Gonçalves Feb 2025

g_m/I_D

$$g_{ms(d)} = \frac{2I_S}{\phi_t} \left(\sqrt{1 + i_{f(r)}} - 1 \right) = \frac{W}{L} \mu C_{ox} n \phi_t \left(\sqrt{1 + i_{f(r)}} - 1 \right)$$

$$g_m = \frac{g_{ms} - g_{md}}{n} \qquad \qquad \frac{g_m}{I_D} = \frac{2}{n\phi_t(\sqrt{1 + i_f} + \sqrt{1 + i_r})}$$

For $V_{DS}/\phi_t <<1$ we have $i_f \simeq i_r$

In saturation $i_f >> i_r$

$$\frac{g_m}{I_D} \cong \frac{1}{n\phi_t\sqrt{1+i_f}}$$

$$\frac{g_m}{I_D} \cong \frac{2}{n\phi_t(\sqrt{1+i_f}+1)}$$



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Intrinsic gain stages: common-source and common-emitter amplifiers





Small-signal circuit and frequency response of the CS and CE amplifiers





Design of the CE and CS amplifiers $|A_v(\omega_u)| = 1$ $g_m = \omega_u C_L = 2\pi \cdot GB \cdot C_L$

BJT in the direct active region

 $I_C = I_S e^{V_{BE}/\phi_t} \qquad \Longrightarrow \qquad I_C = g_m \phi_t = 2\pi \cdot GB \cdot C_L \cdot \phi_t$

MOSFET in saturation

$$I_D = ng_m\phi_t \frac{\left(\sqrt{1+i_f}+1\right)}{2} = ng_m\phi_t \left[1 + \frac{g_m}{2\mu C_{ox}\phi_t(W/L)}\right]$$

Aspect ratio vs. current excess in MOSFET design



REFERENCES

- A. I. A. Cunha, M. C. Schneider and C. Galup-Montoro, "Derivation of the Unified Charge Control Model and Parameter Extraction Procedure", Solid-State Electronics, March 1999.
- O. C. Gouveia Filho, A. I. A. Cunha, M. C. Schneider and C. Galup-Montoro, "Advanced compact model for short-channel MOS transistors", IEEE Custom Integrated Circuits Conference, Orlando, FL, USA, May 2000.
- C. Galup-Montoro and M. C. Schneider, *MOSFET Modeling for Circuit Analysis and Design*, World Scientific, 2007.
- C. M. Adornes, D. G. Alves Neto, M. C. Schneider and C. Galup-Montoro, " Bridging the Gap between Design and Simulation of Low-Voltage CMOS Circuits," Journal of Low Power Electronics and Applications, June 2022.
- D. G. Alves Neto *et al,* "Design-oriented single-piece 5-DC parameter MOSFET model," *IEEE Access*, 2024.

Appendix: the exact model of the longchannel MOSFET¹

$$I_D = -\frac{W}{L} \int_{V_S}^{V_D} \mu Q_I (V_C) d V_C$$

Consequently, the **exact** expressions for g_{ms} and g_{md} are

$$g_{md} = \frac{dI_D}{dV_D} = -\frac{W}{L}\mu Q_D = \frac{2I_S}{\phi_t}q_D$$

$$g_{ms} = -\frac{dI_D}{dV_S} = -\frac{W}{L}\mu Q_S = \frac{2I_S}{\phi_t}q_S$$

¹ H. C. Pao and C. T. Sah, 'Effects of diffusion current on characteristics of metal-oxide (insulator)-semiconductor transistors" Solid-State Electronics, Oct 1966 Bento Gonçalves Feb 2025 28









Deni Germano Alves Neto

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"scan me"

Outline

- ACM2 :
 - Parameter Extraction
 - VT0, IS and n
 - Sigma & Zeta
 - DC characteristics
 - Transconductances in Saturation
 - Example: Inverter CMOS
- Overview of Open-source environment
 - Automatic parameter extraction

ACM2: A simple 5-DC-parameter MOSFET model

$$V_{P} = \frac{V_{GB} - V_{T0} + \sigma(V_{DB} + V_{SB})}{n}$$

$$\frac{V_{P} - V_{SB}}{\phi_{t}} = q_{s} - 1 + \ln(q_{s})$$
Used to calculate q_{s}

$$q_{dsat} = q_{s} + 1 + \frac{1}{\zeta} - \sqrt{\left(1 + \frac{1}{\zeta}\right)^{2} + \frac{2q_{s}}{\zeta}}$$
Used to calculate q_{d}

$$\frac{V_{DS}}{\phi_{t}} = q_{s} - q_{d} + \ln\left(\frac{q_{s} - q_{dsat}}{q_{d} - q_{dsat}}\right)$$

$$I_{D} = I_{S} \frac{(q_{s} + q_{d} + 2)}{1 + \zeta(q_{s} - q_{d})}(q_{s} - q_{d})$$
Specific current I_{S} (W,L)
Threshold voltage V_{T0} (W,L)
$$I_{D} = I_{S} \frac{(q_{s} + q_{d} + 2)}{1 + \zeta(q_{s} - q_{d})}(q_{s} - q_{d})$$

3PM-ACM model in a nutshell

$$I_{D} = I_{S}[i_{f} - i_{r}] \quad \text{where} \quad I_{S} = \mu C_{ox} n \frac{\phi_{t}^{2} W}{2} = I_{SH} \frac{W}{L}$$

$$\frac{V_{P} - V_{S(D)B}}{\phi_{t}} = \sqrt{1 + i_{f(r)}} - 2 + \ln\left(\sqrt{1 + i_{f(r)}} - 1\right) \qquad V_{P} \cong \frac{V_{GB} - V_{T0}}{n} \quad \text{DC eqs}$$
If we choose $i_{f} = 3 \quad \text{Prometry} \quad V_{GB} = V_{T0}$

$$\frac{V_{DS}}{\phi_{t}} = \sqrt{1 + i_{f}} - \sqrt{1 + i_{r}} + \ln\left(\frac{\sqrt{1 + i_{f}} - 1}{\sqrt{1 + i_{r}} - 1}\right)$$

$$g_{ms(d)} = \frac{2I_{S}}{\phi_{t}} \left(\sqrt{1 + i_{f(r)}} - 1\right) \quad \text{Prometry} \quad \frac{W}{L} = \frac{g_{ms(d)}\phi_{t}}{2I_{SH}(\sqrt{1 + i_{f(r)}} - 1)}$$

$$g_{m} = \frac{g_{ms} - g_{md}}{n} \qquad \frac{g_{m}}{I_{D}} = \frac{d(\ln I_{D})}{dV_{G}} = \frac{2}{n\phi_{t}(\sqrt{1 + i_{f}} + \sqrt{1 + i_{r}})}$$
Small-signal





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ACM2 : Transconductances in saturation

$$I_{Dsat} = \frac{2I_{S}}{\zeta} q_{dsat} \quad \text{where} \quad \left\{ \begin{array}{l} I_{S} = \mu C_{ox} n \frac{\Phi_{t}^{2}}{2} \frac{W}{L} = I_{SH} \frac{W}{L} \\ q_{dsat} = q_{s} + 1 + \frac{1}{\zeta} - \sqrt{\left(1 + \frac{1}{\zeta}\right)^{2} + \frac{2q_{s}}{\zeta}} \end{array} \right.$$

$$Definitions \quad \left\{ \begin{array}{l} g_{m} \triangleq \frac{\partial I_{Dsat}}{\partial V_{G}} & g_{d} \triangleq \frac{\partial I_{Dsat}}{\partial V_{D}} & g_{msat3} \triangleq \frac{\partial^{3} I_{Dsat}}{\partial V_{G}^{3}} \end{array} \right.$$

Transconductances in terms of q_s :

Extraction of σ

Common-Source Intrinsic-Gain method


ζ extraction

- $i_{dsat} = \frac{2}{\zeta} q_{dsat} \qquad q_s = \sqrt{1 + \frac{2}{\zeta} q_{dsat}} 1 + q_{dsat}$ $\int \zeta = \frac{2(q_s + 1 \sqrt{1 + i_{dsat}})}{i_{dsat}}$
- q_s calculated using parameters (V_{T0} , n, σ) and UCCM.

$$V_P = \frac{V_{GB} - V_{T0} + \sigma(V_{DB} + V_{SB})}{n}$$

$$\frac{V_P - V_{S(D)B}}{\phi_t} = q_s - 1 + \ln(q_s)$$



Measure $I_{Dsat} = I_D(V_G = V_D = V_{DDmax} \text{ and } V_S = V_B)$ $i_{dsat} = I_{Dsat}/I_S.$



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Qucs-S: Quite universal circuit simulator with SPICE



Qucsator

Qucs-S: Quite universal circuit simulator with SPICE



ACM2¹ vs PSP – 130 nm SiGe IHP² $I_D vs V_{GB}$



Characteristics of a LVT NMOS bulk transistor with W /L = 10μ m/ 120 nm.

¹ACM2 : implemented in verilog-A, compiled by OPENVAF, simulated in Ngspice ² Institut for High-Performance Microelectronics (IHP) open-source PDK

5PM-ACM2 : Transconductance gm, gm3 and gds



CMOS Inverter in 130 nm bulk VTC and short-circuit current





CMOS Inverter in 130 nm bulk Output Voltage and pull-down current



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Open-Source IC design

- Chipathon SSCS 2021 :)
- Analog-front-end for Biosignals AFEbio
- Chipathon-SSCS 2023 & UNIC-CASS 2023
 - Analog IC design
- Mentor in Universalization of IC Design from 2024
 CASS (UNIC-CASS)







Open-Source Design of Integrated Networks





Let's go to Qucs-S!

Useful links:

- Qucs-S oficial page: <u>https://ra3xdh.github.io/</u>
- Qucs-S iterative doc: <u>https://qucs-s-help.readthedocs.io/</u>
- Ngspice oficial page : <u>https://ngspice.sourceforge.io/index.html</u>
- Google Colab: ACM2 & LNA design:

https://colab.research.google.com/drive/1s3PKF6pf3zIhlTj6jcqhCLIcGfJ_UEE?usp=sharing



Github – ACM2

Github - Content

ACMmodel Merge pull request #26 from gabrielm	aranhao/main 🚥	71ee7cd · 6 months ago
Examples	Update SKY130 and GF180 using ACM ex	amples on xschem
Verilog-A	Update PMOS_ACM_2V0.va	
docs	Delete 5PM_NewCAS.pdf	
	Update LICENSE	
C README.md	Update README.md	
C README I ECL-2.0 license		

Advanced Compact MOSFET model (ACM)

ACM is a simple MOSFET model to design and simulate Analog, Mixed-Signal, and RF circuits

Examples of PDKs and circuit simulators using the ACM model







Verilog-A code Available!

DSFET_model / Verilog-A / NMOS_ACM_2V0.va				
🔶 ACN	Mmodel Update NMOS_ACM_2V0.va			
Code	Blame 291 lines (245 loc) · 12.3 KB			
	//*************************************	**		
	// * ACM NMOS model (Verilog-A)			
	// * 07/2023 V2.0.0			
	//*************************************	**		
	// ************************************	**		
	<pre>// * Copyright under the ECL-2.0 license</pre>			
	// * Universidade Federal de Santa Catarina			
	// *			
	// * Current developers: Deni Germano Alves Neto (Doctoral student, UFSC)			
11	<pre>// * Cristina Missel Adornes (Doctoral student, UFSC)</pre>			
12	// * Gabriel Maranhao (Doctoral student, UFSC)			
13	// *			
	<pre>// * Project Supervisors: Prof. Carlos Galup-Montoro</pre>			
	// * Prof. Marcio Cherem Schneider			
	// ************************************	**		
17				
	`include "constants.vams"			
	`include "disciplines.vams"			
	<pre>// function of the algorithm 443 to calculate de normalize charge densities</pre>			
	`define algo_443(Z,qn) \			
	if(Z < 0.7385) begin \			
	numeratorD = Z + (4.0/3.0)*Z*Z; \			
	denominatorD = 1.0 + (7.0/3.0)*Z+(5.0/6.0)*Z*Z; \			
	WnD = numeratorD/denominatorD; \			
	end else begin \			
	numeratorD = ln(Z)*ln(Z)+2.0*ln(Z)-3.0; \			
	denominatorD = 7.0*ln(Z)*ln(Z) + 58.0*ln(Z) +127.0; \			
30	WnD = ln(Z) - 24.0*(numeratorD/denominatorD); \			







Part 3 Analog IC design





Márcio Cherem Schneider

https://github.com/ACMmodel/MOSFET_model

Outline

Analog circuit design

- 1. Current mirrors
- 2. High-swing cascode current mirror
- 3. Current gain schemes
- 4. Current and voltage references
- 5. Single-stage amplifiers

ACM2 applied to weak inversion

ACM2 parameters: V_{T0} , I_S , n, σ , ζ $q_{S(D)} = Q_{S(D)}/(-nC_{ox}\phi_t)$ $q_{S(D)} = e^{\left(\frac{V_P - V_{S(D)B}}{\phi_t} + 1\right)}$ UCCM $I_D/I_S = i_f - i_r = (I_F - I_R)/I_S$

$$V_{P} = \frac{V_{GB} - (V_{T0} - \sigma(V_{DB} + V_{SB}))}{n}$$
$$i_{D} = I_{D} / I_{S} = 2(q_{S} - q_{D})$$

$$I_{S} = \frac{W}{L} \mu_{s} n C_{ox} \frac{\phi_{t}^{2}}{2} = SI_{SH}$$

normalization (specific) current

WI : *q*_{*S(D)} << 1</sub>*

ratio of diffusion-related velocity to saturation velocity

1. Current mirrors



What's this saturation voltage V_{Dsat}? No clear definition.

An approximation for the saturation voltage:

$$V_{DSsat} \cong \phi_t \left(\sqrt{1 + i_f} + 3 \right)$$

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1. Current mirrors



Linear approximation

$$\frac{I_{out}}{I_{in}} = \frac{I_S f (V_G - V_T, V_S) (1 + V_{out} / V_A)}{I_S f (V_G - V_T, V_S) (1 + V_{in} / V_A)}$$



Error due to difference in drain voltages

 V_A is non-physical

A simple case: error for weak inversion using ACM2

$$I_{in} = I_{F1} - I_{R1} = I_{F1} \left[1 - e^{-V_{in}/\varphi_t} \right]$$

$$I_{out} = I_{F2} - I_{R2} = I_{F2} \left[1 - e^{-V_{out}/\varphi_t} \right]$$

$$I_{F2} = I_{F1} e^{\sigma(V_{out} - V_{in})/n\varphi_t}$$

$$\frac{I_{out}}{I_{in}} = e^{\frac{\sigma(V_{out} - V_{in})}{n\varphi_t}} \frac{1 - e^{-V_{out}/\varphi_t}}{1 - e^{-V_{in}/\varphi_t}}$$

$$\frac{I_{out}}{I_{in}} \cong e^{\frac{\sigma(V_{out} - V_{in})}{n\varphi_t}} \text{ for } V_{out} \& V_{in} \gg \varphi_t$$

1. Current mirrors

Dependence of the output current (conductance) on the output voltage





1. Current mirrors Error due to mismatch

$$\frac{\sigma^2(I_D)}{I_D^2} \approx \frac{2}{WL} \left\{ A_{ISH}^2 + \left(\frac{A_{VT}}{n\phi_t}\right)^2 \left[\frac{2}{\left(\sqrt{1+i_f}+1\right)}\right]^2 \right\}$$



Dependence of current matching on inversion level in linear and saturation regions $|V_{DS}| = 20 \text{ mV}$ and $|V_{DS}| = 2 \text{ V}$, respectively, for the large, medium-size, and small PMOS transistor pairs. Statistics determined for 18 pairs of transistors.

2. Cascode current mirrors

Self-biased cascode current mirror (SBCCM)



Node X follows approximately node Y

Assume $M_1 \equiv M_2 \equiv M_3 \equiv M_4$

- If M_4 operates in saturation, then $V_X \cong V_Y$ and $I_{out} \cong I_{in}$
- V_{out} is transferred to V_X with attenuation $(\frac{g_{md4}}{g_{ms4}})$ of common-gate gain
- Input constraint: $V_{DD} > V_{CS,min} + V_{GS3} + V_{GS1}$
- Output constraint: V_{out}> V_{GS1}+V_{DS4,sat} for saturation of M₄ (In most cases, V_{GS}> V_{DS,sat})
- The output conductance:

$$y_o = \frac{i_{out}}{v_{out}} \approx g_{md2} \frac{g_{md4}}{g_{ms4}}$$

SBCCM is not a low-voltage current mirror

2. Cascode current mirrors

Low-voltage cascode current mirror: lower VDD



- Input constraint: V_{DD} > V_{CS,min}+V_{DS3,sat}+V_{GS1}
- Output constraint: $V_{out} > V_{DS2,sat} + \Delta V + V_{DS4,sat}$ for saturation of M_4

Goal: Design V_{G5} to bias M2 (M1) at the edge of saturation $V_{S4} = V_{DS2} = V_{DSsat_2} + \Delta V$ $f2 = i_{f4} = \frac{I_0}{I_{S4}} \cong \frac{I_{in}}{I_{S4}}$ $V_{DSsat} = \varphi_t(\sqrt{1 + i_{f2}} + 3)$ $V_{P4} = V_{S4} + \varphi_t \left[\sqrt{1 + i_{f4}} - 2 + \ln\left(\sqrt{1 + i_{f4}} - 1\right)\right]$ (I) $V_{P5} = \varphi_t \left[\sqrt{1 + i_{f5}} - 2 + \ln\left(\sqrt{1 + i_{f5}} - 1\right)\right]$ (II)

Equate (I) to (II) to find i_{f5} Choose I_{S5} & I_{B5} to comply with $i_{f5} \cong \frac{I_{B5}}{I_{S5}}$

$$I_{S5} = I_{SH} \frac{W}{L}$$
$$y_o = \frac{i_{out}}{v_{out}} \approx g_{md2} \frac{g_{md4}}{g_{ms4}}$$

2. Cascode current mirrors Low-voltage cascode current mirror - 3



•V. C. Vincence, C. Galup-Montoro and M. C. Schneider, "<u>A high-swing MOS cascode bias circuit</u>," *IEEE Trans. Circuits Syst. II*, vol. 47, no. 11, pp. 1325-1328, Nov. 2000.

• P. Aguirre and F. Silveira, "Bias circuit design for low-voltage cascode transistors," *Proc. of SBCCI 2006*, pp. 94-98, Sep. 2006.



3. Current gain schemes

Example: Gain-of-16 current mirror



4. Current and voltage references The self-cascode MOSFET (SCM)



$$I_X = I_{S2}i_{f2} = I_{S1}(i_{f1} - i_{f2})$$

$$I$$

$$i_{f1} = \left(1 + \frac{I_{S2}}{I_{S1}}\right) = \left(1 + \frac{S_2}{S_1}\right)i_{f2} = \alpha i_{f2}$$

Applying UICM to M1

$$\frac{V_X}{\phi_t} = \sqrt{1 + \alpha i_{f2}} - \sqrt{1 + i_{f2}} + \ln\left(\frac{\sqrt{1 + \alpha i_{f2}} - 1}{\sqrt{1 + i_{f2}}}\right)$$

 V_X is PTAT if i_{f2} is independent of temperature In other words, V_X is PTAT if i_{f2} is a scaled replica of I_S

In weak inversion,

$$i_{f1} = \alpha i_{f2} \ll 1$$

$$\rightarrow \ln \alpha \implies V_X \text{ is PTAT and independent} \\ \text{of the current coefficient}$$

4. Current and voltage references The self-cascode MOSFET (SCM)



C. Rossi, C. Galup-Montoro and M.C. Schneider, <u>"PTAT</u> voltage generator based on an MOS voltage divider", Proceedings of Nanotech 2007, pp. 626- 629, May 2007.

4. Current and voltage references The self-cascode MOSFET (SCM)





4. Current and voltage references A self-biased current source (SBCS)



If M_3 and M_4 operate in WI :

$$\ln \alpha_{34} = \sqrt{1 + \alpha_{12}i_{f2}} - \sqrt{1 + i_{f2}} + \ln\left(\frac{\sqrt{1 + \alpha_{12}i_{f2}} - 1}{\sqrt{1 + i_{f2}} - 1}\right);$$
$$\alpha_{12} = 1 + \frac{S_2}{S_1} \qquad \alpha_{34} = 1 + \frac{S_4}{S_3}$$

Thus, i_{f2} (the inversion level of M₂) is constant (it depends only on the geometrical ratios α_{12} and α_{34})

The reference current $I_{B2} = I_{S2}i_{f2}$ is proportional to the specific current of M₂. $I_{S2} = I_{SH}(W/L)_2 = I_{SH}S_2$

 Stability at the operating point: Negative feedback>positive feedback

4. Current and voltage references MOS-only voltage reference





E. Vittoz and O. Neyroud, "A low-voltage CMOS bandgap reference," Solid-State Circuits, IEEE Journal of, vol. 14, no. 3, pp. 573–579, 1979.

5. Single-stage amplifiers Common-source amplifier The intrinsic gain



The gain is a function of technology, transistor channel length, bias current, and output voltage

5. Single-stage amplifiers Common-source amplifier

Velocity saturation not accounted for :





DIBL coefficient versus channel length



J. H. Huang et al., A physical model for MOSFET output resistance, IEDM 1992
5. Single-stage amplifiers

Push-pull amplifier (static CMOS inverter)



For "deep" saturation:

$$\frac{1}{A_V} = -\left(\frac{g_{m1}\sigma_1}{g_{m1} + g_{m2}} + \frac{g_{m2}\sigma_2}{g_{m1} + g_{m2}}\right)$$

$$A_V = -\frac{2}{\sigma_1 + \sigma_2} \text{ for } g_{m1} = g_{m2}$$

5. Single-stage amplifiers Common-gate amplifier





UICM

$$V_{TH} = \frac{V_{bias} - V_{T0N}}{n_N} - \phi_t \left[\sqrt{1 + i_{f1}} - 2 + \ln\left(\sqrt{1 + i_{f1}} - 1\right) \right]$$





THANKS FOR ATTENDING THE

ANALOG SECTION OF THE

TUTORIAL

QUESTIONS?





Part 4

Advanced Compact MOSFET Model: Design Methodology-Application to Low Noise Amplifier





Sylvain Bourdel

https://github.com/ACMmodel/MOSFET_model

https://colab.research.google.com/drive/1s3PKF6pf3zIhlTj6jc-

<u>qhCLIcGfJ_UEE?usp=sharing#scrollTo=EMGo7auzwwkW</u>

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- Overview of Design Methods for RFIC
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Overview of Design Methods for RFIC General Principle

• 1st Step: Architecture Choice





• 2nd Step: Circuit Analysis





$$|G_T| = |G_v|Q_{IN} = \frac{(G_m R_F - 1)R_O}{(R_O + R_F)}\sqrt{1 + Q_P^2}$$

Overview of Design Methods for RFIC General Principle

• 3rd Step: Circuit Sizing



Overview of Design Methods for RFIC General Principle

• 3rd Step: Circuit Sizing



Region based model

sat.
$$I_d = 2.K_n \frac{W}{L} \left[\left(V_{gs} - V_t \right) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

lin.
$$I_d = K_n \frac{W}{L} \left(V_{gs} - V_t \right)^2 \left(1 + \frac{k_{en}}{L} V_{ds} \right)$$

PROS: simple CONS: Inaccurate with short channel MOS

Overview of Design Methods for RFIC General Principle

• 3rd Step: Circuit Sizing



BSIM3 / UTSOI compact model PROS:

- Accurate
- Direct relationship between requirements and physical parameters CONS:
- Increases the gap between Physic and design
- Long optimization

Overview of Design Methods for RFIC New Trends

• Issue in the sizing step



- How to maintain simplicity and accuracy with the scaling and especially Short Channel Effects (SCE) at the sizing step.
- A possible solution
 - g_m/ld design approaches
 - LUT based
 - ACM or EKV based

Overview of Design Methods for RFIC g_m /Id approach based on LUT

General Principle

[Silveira, Flandre, Jespers – JSSC 1996]

• For a given structure



Overview of Design Methods for RFIC g_m /Id approach based on LUT



Overview of Design Methods for RFIC g_m/ld approach based on LUT



- For a given structure
- Methods are based on ٠
 - \Rightarrow Extraction of MOS parameters (LUT)
 - $I_D/(W/L) vs g_m/I_D$
 - $g_m/I_D vs V_{gs0}$
 - \Rightarrow Set of equations

$$SR = \frac{B \cdot I_{D1}}{C_L}$$
$$f_T = \frac{B \cdot g_{m1}}{2\Pi \cdot C_L}$$
(5)

$$A_{0} = \left(\frac{g_{m}}{I_{D}}\right)_{1} \cdot \left(\frac{g_{m}}{I_{D}}\right)_{2} \cdot \frac{1}{\frac{1}{V_{A6} \cdot V_{A7}} + \frac{1}{V_{A9} \cdot V_{A10}}}$$
(4)

Overview of Design Methods for RFIC g_m/ld approach based on LUT



Overview of Design Methods for RFIC g_m/Id approach based on LUT



Overview of Design Methods for RFIC g_m/Id approach based All Region Models (ACM/EKV)

All Region 3PM model

- EKV [Enz, Krummenacher, Vittoz]
- ACM [Schneider, Galup]
- With a 3PM model we have :



$$\frac{g_m}{I_D} = \frac{2}{n\phi_t \left(1 + \sqrt{1 + i_f}\right)} \qquad g_m = \frac{2I_s}{\phi_t} \left(\sqrt{1 + i_f} - 1\right) \qquad V_P - V_{S(D)} = \phi_t \left[\sqrt{1 + i_{f(r)}} - 2 + \ln\left(\sqrt{1 + i_{f(r)}} - 1\right)\right] \\ V_P \cong \frac{V_G - V_{T0}}{n}$$

- PROS: The sizing is straightforward
- CONS: Inaccurate with SCE

Non-linearities can't be captured gds effect can't be captured

Overview of Design Methods for RFIC g_m/ld in RFIC design

Design Regions



• f_t grows with i_f



Overview of Design Methods for RFIC g_m/Id in RFIC design

Design Regions

• f_t grows with $i_f \underline{but} g_m / I_D$ reduces with i_f



Year 2000 (RF-180nm)

Year 2016 (RF-22nm) up to now



Overview of Design Methods for RFIC g_m/ld in RFIC design

$g_m f_t / I_D$: A First Approach

A. Shameli et P. Heydari, « Ultra-Low Power RFIC Design Using Moderately Inverted MOSFETs: An Analytical/Experimental Study », *RFCI*, 2004.

- For RF design
- A FOM that maximize the gain bandwidth product

$$\frac{g_m f_T}{I_D}$$

- Gives the i_f that produces the best GBW product
- Not optimal :
 - $_{\odot}$ the gain or the bandwidth might be oversized.
 - $\,\circ\,$ Do not depends on the topology



Overview of Design Methods for RFIC g_m/Id in RFIC design

Function Based FoM

- For a given Function
- Define a FoM for the function

Find the *i_f* that maximizes the FoM







LO



I. Song et B.-G. Park, «A Simple Figure of Merit of RF MOSFET for Low-Noise Amplifier Design », *Electron Device Lett. IEEE*, vol. 29(12), 2008.



$$PN(\Delta f) = 10 \log \left(\frac{k_B T \pi^2}{64Q^2} \frac{g_m}{I_D} \frac{1}{I_D} \frac{f_0}{\Delta f)^2} \lambda \right)$$

R. Fioreli ; J. Núñez ; F. Silveira; "All-Rnversion region gm/ID methodology for RF circuits in FinFET technologies" 2018 NEWCAS

Overview of Design Methods for RFIC g_m/Id in RFIC design

Circuit Based Method

Circuit Sizing Based on LUT

- For a given Circuit
- Similar to the one introduced by Silveira, Flandre, Jespers in 1996
- Capacitance and noise effects can be captured in LUT for bandwidth and noise analysis
- Well known and well referenced
- But ...
- Still complicated
- The gap between physics and design remains
- · No analytical relationships to size « by hand »



RESEARCH ARTICLE

Circuit Sizing Based on ACM

- Equations can be derived for a given circuit
- Quasi analytical approach
 - R-F LNA designed with 3PM-ACM [Bourdel ICECS 2019]
- Gds and gm3 (NL) are taken into account
 - R-F LNA designed with 7PM-ACM [Bouchoucha ISCAS
 - 202BNA designed with 5PM-ACM2[Alves Neto ACCESS 2024]



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LNA Design considerations Noise Factor

• NF is the signal to noise ratio (SNR) degradation (Eq. 1)



• NF also express the quantity of noise added by the stage regarding the noise delivered by the source (R_g) on Δf . (Eq. 2)

LNA Design considerations Friss Formula

≻RF Basis



In RFIC, impedances are complex and never equal. This formula cannot be us However, the following statements remain valid in RFIC.

The gain of the first stages reduces the NF of the following ones.



=> The receiver chain must start by amplifiers
=> The NE of the first stage must be as low as page

=> The NF of the first stage must be as low as possible Note that lossy stages (G_v <1) increase the NF, especially when they are in front

of the receiver chain (antenna filter, image rejection filter, antenna switch).

LNA Design considerations Input patching

System point of vue

- LNA is the first device due to Friss formula
- Antenna is mainly 50 Ω .



In CMOS

- Generally, the input impedance is capacitive Re (Y11) is very low
- For exemple : CS amplifier



LNA Design considerations Non-Linearities



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Resistive Feedback LNA Topology

R-Feedback general considerations

- Simple
- Compact (No-inductors)
- Wideband
- RF allows to synthesize a real part in Zin
- LG and CGS' will help in cancelling the imaginary part
- while controlling Qin

Input impedance

$$V_{BF} \xrightarrow{L_G} V_I \xrightarrow{V_I} \xrightarrow{M(W,L)} \xrightarrow{C_L} C_L$$



$$R_O = \frac{R_L r_{DS}}{R_L + r_{DS}}$$

$$\Re(Z_{IN}) = R_S = \frac{R_P}{(1+Q_P^2)} = 50 \ \Omega \qquad Q_P = R_P C_T \omega_0 \qquad C_T = C_{GS} + C'_{GS} + C_P$$

$$\Im(Z_{IN}) = 0 = L_G s + \frac{Q_P^2}{C_T s (1+Q_P)^2} \qquad \text{Note:} \qquad \\ \Re(Zp) = \frac{1 + \left(\frac{f}{f_c}\right)^2}{\frac{1+G_m R_O}{R_O + R_F} + \frac{1}{R_F} \left(\frac{f}{f_c}\right)^2}, \qquad \\ \Re(Zp) = -\left(1 + \frac{R_F}{R_O}\right)^2 \frac{1 + \left(\frac{f}{f_c}\right)^2}{2\pi f C_L(G_m R_F - 1)} \qquad \\ \frac{f_C}{f} = N > 3 \qquad \\ 103$$

Resistive Feedback LNA Topology



Noise Figure

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What we need in ACM-2 Small signal parameters

Small signal (AC) parameters

•
$$G_{T}$$
, NF, IIP3 vs (G_{m}, G_{m3}, G_{DS})
 $g_{m} = \frac{\partial I_{D}}{\partial V_{G}} = \frac{I_{S}}{\phi_{t}}g_{G} = \frac{I_{S}}{n\phi_{t}}\frac{2(q_{s} - q_{d}) - i_{d}\zeta\left(\frac{q_{s}}{1 + q_{s}} - \frac{q_{d}}{1 + q_{d}}\right)}{1 + \zeta\left(q_{s} - q_{d}\right)}$
 $g_{ds} = \frac{I_{S}}{\phi_{t}}g_{D} = \frac{I_{S}}{n\phi_{t}}\frac{2(q_{s}\sigma - q_{d}(\sigma - n)) - i_{d}\zeta\left(\sigma\frac{q_{s}}{1 + q_{s}} - (\sigma - n)\frac{q_{d}}{1 + q_{d}}\right)}{1 + \zeta\left(q_{s} - q_{d}\right)}$

$$g_{m3} = \frac{I_{S}}{(n\phi_{t})^{3}} \left\{ \frac{\frac{2q_{s}}{(1+q_{s})^{3}} - \frac{2q_{d}}{(1+q_{d})^{3}} - \zeta n^{2}g_{G2}\left(\frac{q_{s}}{1+q_{s}} - \frac{q_{d}}{1+q_{d}}\right) - \zeta \left[2ng_{G}\left(\frac{q_{s}}{(1+q_{s})^{3}} - \frac{q_{d}}{(1+q_{d})^{3}}\right) + i_{d}\left(\frac{q_{d}(1-2q_{d})}{(1+q_{d})^{5}} - \frac{q_{d}(1-2q_{d})}{(1+q_{d})^{5}}\right)\right]}{1 + \zeta \left(q_{s} - q_{d}\right)} \right\}$$

- Valid in all region (qs and qd shall be explored)
- We consider only saturation

$$q_{dsat} = q_s + 1 + \frac{1}{\zeta} - \sqrt{\left(1 + \frac{1}{\zeta}\right)^2 + \frac{2q_s}{\zeta}}$$

$$g_{msat} = \frac{2I_S}{n\phi_t} \frac{q_s}{1 + \zeta(q_s + 1)} \qquad g_{dsat} = \frac{\sigma}{n} \frac{2I_S}{\phi_t} \frac{q_s}{1 + \zeta(q_s + 1)} \qquad g_{msat3} = \frac{16I_S}{(n\phi_t)^3} \frac{q_s}{(q_s + 1)^3} \frac{2 - 2\zeta q_s - 3\zeta q_s^2}{(q_s + 1)^4}$$

• It reduces the exploration to only qs.

What we need in ACM-2 Small signal parameters

Large signal (DC) parameters

• To compute the final voltages

$$V_T = V_{T0} - \sigma(V_{SB} + V_{DB})$$
$$V_P = \frac{V_{GB} - V_T}{n}$$
$$\frac{V_P - V_{S(D)B}}{\phi_t} = q_{s(d)} - 1 + \ln q_{s(d)}$$

• Sometime we'll use i_f in the code

$$i_{d} = i_{f} - i_{r} \quad \longleftrightarrow \quad q_{S(D)} = \sqrt{1 + i_{f(r)}} - 1, \qquad \longleftrightarrow \quad i_{d} = \frac{(q_{s} + q_{d} + 2)}{1 + \zeta |q_{s} - q_{d}|} (q_{s} - q_{d})$$

$$I_{D} = I_{S} \cdot i_{d}$$

$$I_{S} = \frac{\mu C'_{ox} n (U_{T})^{2} W}{2}$$

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General Approach :

Requirements

- GT, NF, IIP3, IDC
- fo, CL, BW
 No Tradeoff

Design parameters

- ACM parameters for a fixed L
- $V_{T0}, I_S, n, \sigma, \zeta$

Design Variables (parameters)

- W, qs are first order design variables (qs = inversion level sets the energy efficiency and the voltages)
- RL, RF are second order design variables

Approach:

- Explore the different tradeoff (the design space) on GT, NF, IIP3 and IDC by playing with W, qs.
- In saturation region (only qs is needed)

$$g_{msat} = \frac{2I_S}{n\phi_t} \frac{q_s}{1 + \zeta(q_s + 1)} \qquad g_{msat3} = \frac{16I_S}{(n\phi_t)^3} \frac{q_s}{(q_s + 1)^3} \frac{2 - 2\zeta q_s - 3\zeta q_s^2}{(\zeta q_s + 2)^4} \qquad g_{dsat} = \sigma \frac{2I_S}{n\phi_t} \frac{q_s}{1 + \zeta(q_s + 1)^3} \frac{q_s}{(\zeta q_s + 2)^4}$$

• Circuit equations depend on **G**m and ACM gives **g**m (normalized)...

$$|G_{T}| = \frac{(G_{m}R_{F} - 1)R_{O}}{(R_{O} + R_{F})} \sqrt{\frac{R_{O} + R_{F}}{R_{S}(1 + G_{m}R_{O})}} \quad G_{[m;DS]x} = \mu C'_{ox} \frac{(U_{T})^{2-x}}{2} \frac{W}{L} g_{[m;DS]x} \qquad I_{S} = \mu C'_{ox} n \frac{(U_{T})^{2}}{2} \frac{W}{L} \quad I_{SL} = \frac{I_{S}}{W} = \mu C'_{ox} n \frac{(U_{T})^{2}}{2L} \frac{W}{L} = \frac{I_{S}}{W} = \frac{I_{S}}{$$

Reducing the variables :

 $|G_{T}| = \frac{(G_{m}R_{F} - 1)R_{O}}{(R_{O} + R_{F})} \sqrt{\frac{R_{O} + R_{F}}{R_{S}(1 + G_{m}R_{O})}} \quad G_{T}(G_{m}; R_{0}; R_{F}; R_{S}) = G_{T}(W; q_{s}; I_{D}; V_{DSAT}; V_{T0}; I_{S}; n; \sigma; \zeta; f_{c}; C_{L})$ Starting from : variables parameters $G_{[m;DS]x} = \mu C'_{ox} \frac{(U_T)^{2-x} W}{L} g_{[m;DS]x} \longrightarrow g_{msat} = \frac{2I_S}{n \phi_1} \frac{(q_s)}{1 + \zeta(q_s + 1)} \longrightarrow G_m(q_S; W)$ $V_{\mathcal{B}_{\mathcal{C}}} \stackrel{L_{\mathcal{G}}}{\longrightarrow} V_{\mathcal{I}} \stackrel{V_{\mathcal{I}}}{\longrightarrow} \mathcal{I}_{\mathcal{I}} \stackrel{V_{\mathcal$ $\implies g_{dsat} = \frac{\sigma}{n} \frac{2I_S}{\phi_c} \frac{q_s}{1 + \zeta(q + 1)} \implies G_{DS}(q_S; W)$ $\implies R_0(q_S; W)$ $R_F = \frac{R_0}{2\pi R_0 C_L f_c - 1} \qquad \Longrightarrow \qquad R_F(q_S; W) \Big|_{f_c}$ $G_T(G_m; R_0; R_F; R_S) = G_T(W; q_S)$ $\left(f_c = \frac{R_O + R_F}{2\pi R_O R_F C}\right)$

Finally: $I_D(q_s; W)$; $R_F(q_s; W)$; $R_0(q_s; W)$; $G_T(q_s; W)$

 $I_D(q_s; W)$







Computing GT, F and IIP3:





Setting the final value:

- We choose W, that gives q_S for a given G_T .
- (W; q_S) gives R_0 , R_L , I_D , G_m , V_{DSAT} and V_G

Input Matching

- With R_0 , R_F , $G_m => R_P$ $\Re(Zp) = R_P = \frac{R_O + R_F}{1 + G_m R_O}$,
- With R_S and R_P calculate Q_P $\Re(Z_{IN}) = R_S = \frac{R_P}{(1+Q_P^2)} = 50 \Omega$

• With Q_P calculate C_T and C_{GS} ', $Q_P = R_P C_T \omega_0$, $C_T = C_{GS} + C'_{GS} + C_P$, $C_P = \frac{R_O^2 C_L (G_m R_F - 1)}{(R_O + R_F)^2}$

• With C_T calculate L_G $\Im(Z_{IN}) = 0 = L_G s + \frac{Q_P^2}{C_T s (1+Q_P)^2}$



Now, let's simulate with a real PDK

