The Advanced Compact MOSFET (ACM) Model for Circuit Analysis and Design

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- The Pao-Sah “exact” MOSFET model
- The capacitive model of the field-effect
- The ACM (charge–based) model
- Design oriented (current-based) model
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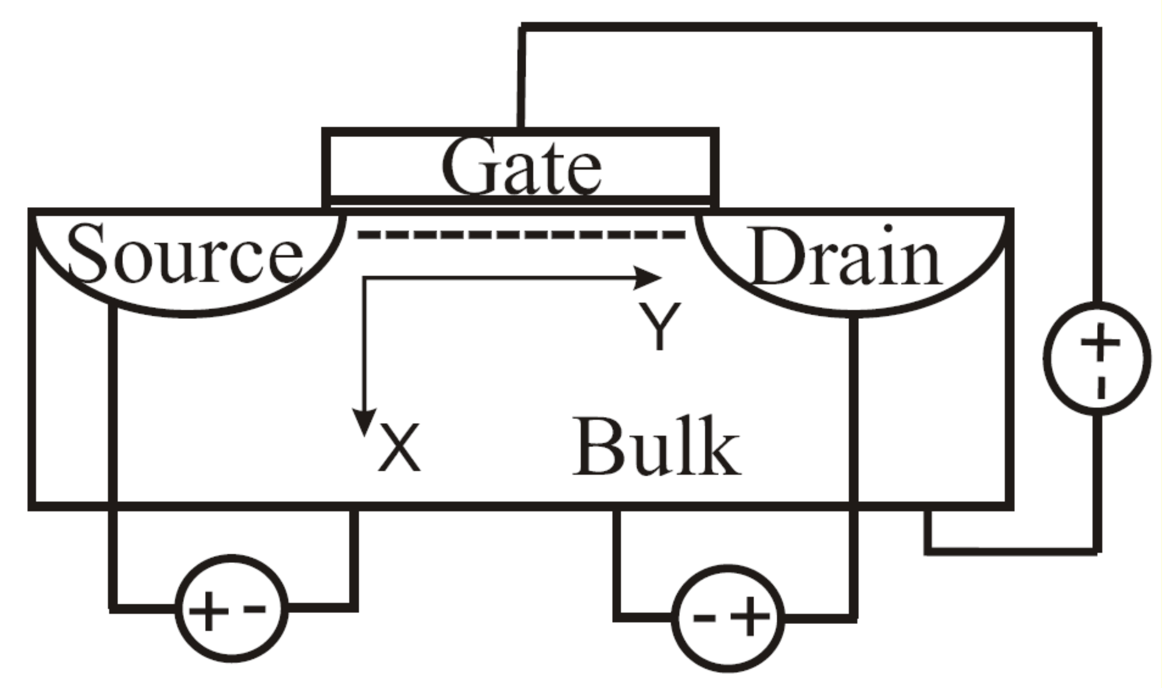
The Pao-Sah model-1: MOSFET operation

2-D problem separated into two 1-D problems:



Vertical 1-D field electrostatics control conduction charge

Longitudinal 1-D field controls current flow

The Pao-Sah model-2

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

⇓

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q_I'\right) dV_C$$

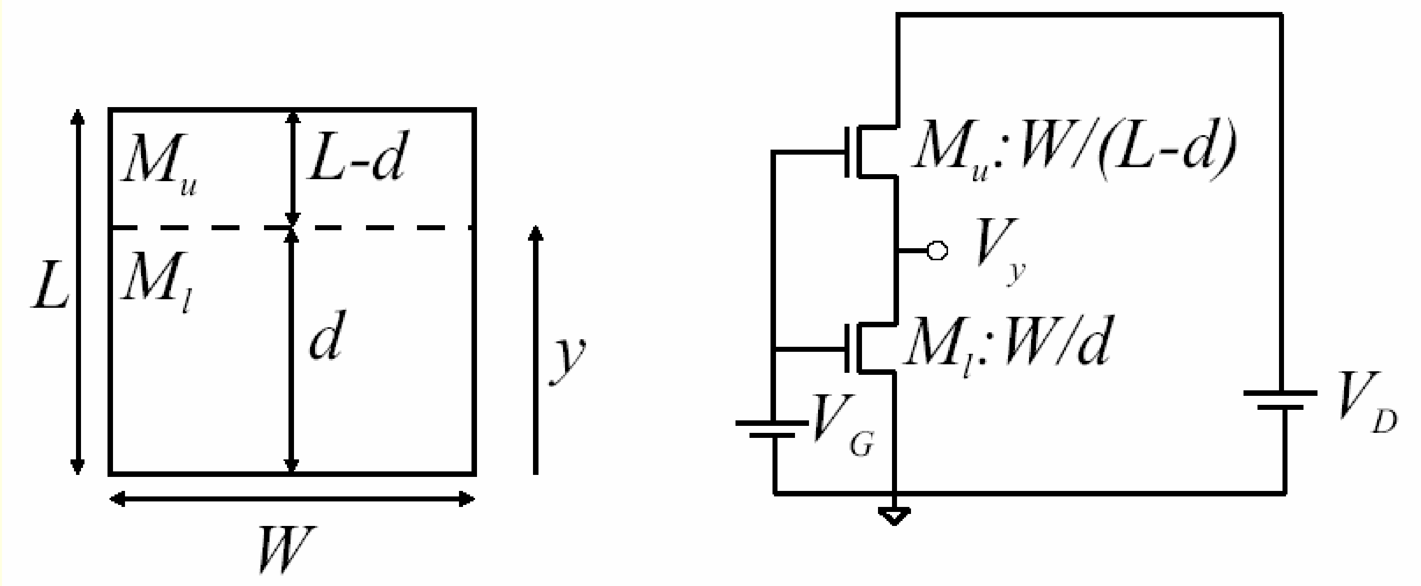
μ: carrier mobility
W : channel width
Q'_I : inversion charge density
$V_S \le V_C \le V_D$,
y: distance along the channel

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G, V_S} = -\frac{W}{L}\mu Q_I'(V_D, V_G)$$

⇑

valid from weak inversion to strong inversion

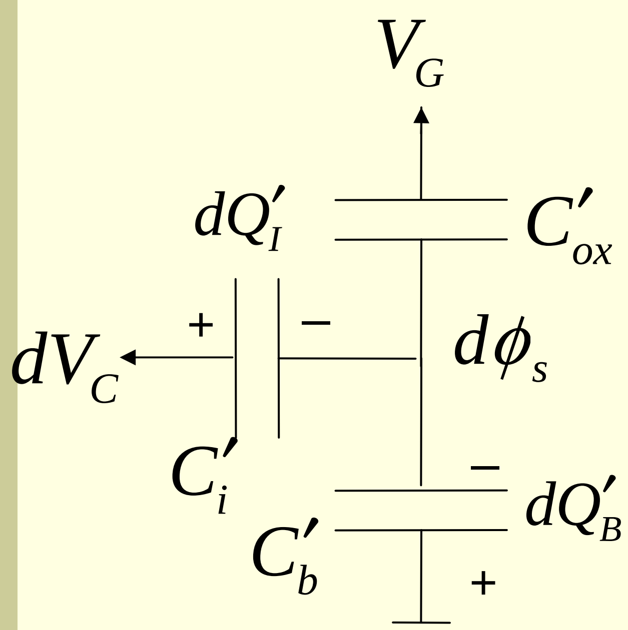
The Pao-Sah model-3:Consistency for the series association



Virtual cut of a transistor into two parts

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q_I'\right)dV_C = \frac{W}{d}\int_{V_S}^{V_Y} \mu\left(-Q_I'\right)dV_C = \frac{W}{L-d}\int_{V_Y}^{V_D} \mu\left(-Q_I'\right)dV_C$$

Capacitive model of the field effect



$$C'_i = -\frac{dQ'_I}{d\phi_s} \cong -\frac{Q'_I}{\phi_t}$$

$$dQ'_I = C'_i(dV_C - d\phi_s)$$

$$dV_C = d\phi_s - \frac{\phi_t}{Q'_I}dQ'_I$$

$$I_D = -\mu W Q'_I \frac{dV_C}{dy} = -\mu W Q'_I \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ'_I}{dy}$$

drift diffusion

The ACM model-1: Linearization of the depletion charge variation

Charge sheet approximation of the inversion charge

$$Q'_I = -C'_{ox}\left(V_G - V_{FB} - \phi_s\right) - Q'_B$$

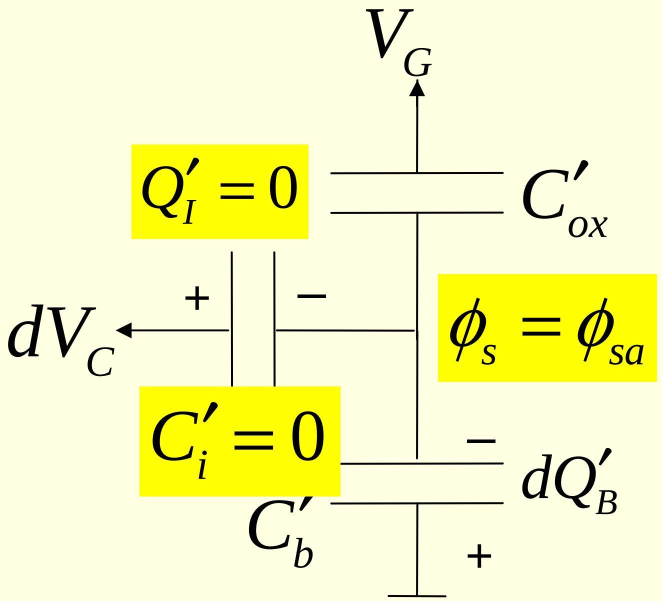
- For constant V_G, it follows that

$$dQ'_I = C'_{ox}d\phi_s - dQ'_B = (C'_{ox} + C'_b)d\phi_s = nC'_{ox}d\phi_s$$

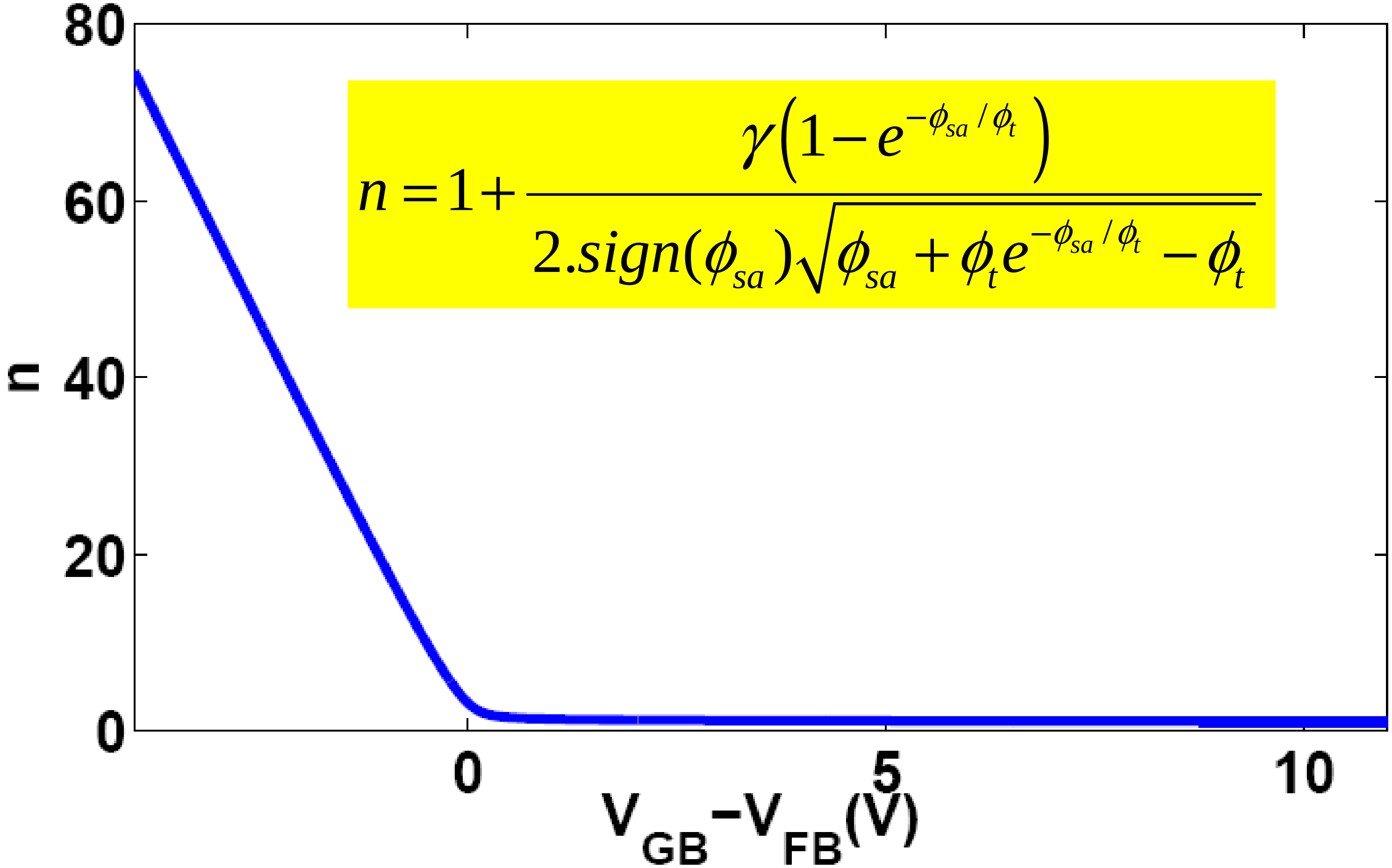
$$n = 1 + \frac{C'_b(V_G)}{C'_{ox}} = n(V_G)$$

$$\frac{1}{n} = \frac{d\phi_{sa}}{dV_G} = \frac{C'_{ox}}{C'_{ox} + C'_b}$$

$$\phi_{sa} = \phi_s\Big|_{Q'_I=0}$$

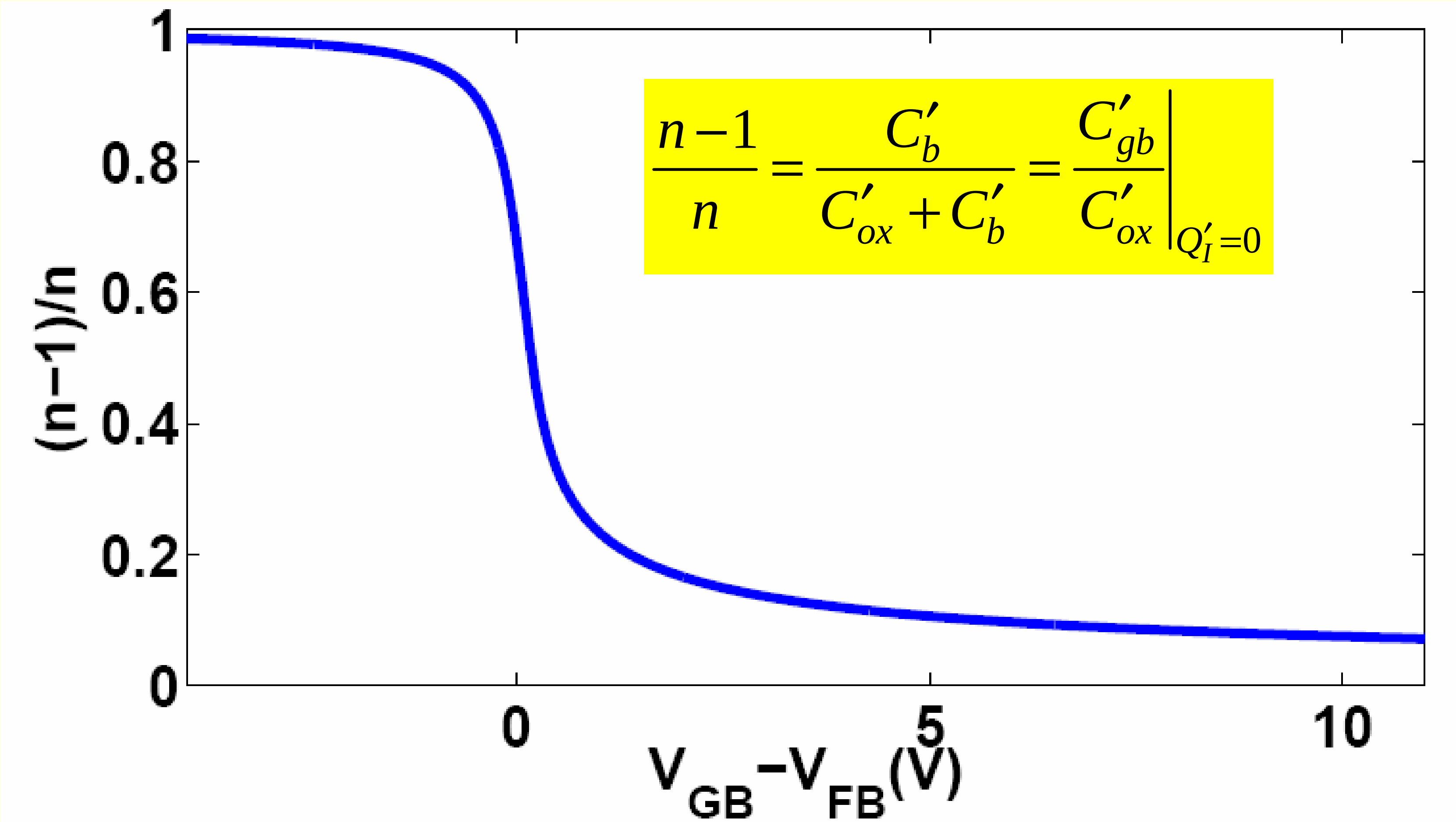


The ACM model-2: Slope factor *n*



$$n = 1 + \frac{\gamma\left(1 - e^{-\phi_{sa}/\phi_t}\right)}{2.sign(\phi_{sa})\sqrt{\phi_{sa} + \phi_t e^{-\phi_{sa}/\phi_t} - \phi_t}}$$

The ACM model-3: Slope factor *n*



$$\frac{n-1}{n} = \frac{C'_b}{C'_{ox} + C'_b} = \left.\frac{C'_{gb}}{C'_{ox}}\right|_{Q'_I=0}$$

The ACM model-4: I_D formula

$$I_D = -\mu W Q_I' \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ_I'}{dy}$$

$$dQ_I' = nC_{ox}' d\phi_s$$

$$\Rightarrow \quad I_D = \frac{\mu W}{L}\left[\frac{Q_{IS}'^{\,2} - Q_{ID}'^{\,2}}{2nC_{ox}'} - \phi_t\left(Q_{IS}' - Q_{ID}'\right)\right]$$

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G, V_S} = \frac{\mu W}{L}\left[\frac{-Q_{ID}'}{nC_{ox}'} + \phi_t\right]\frac{dQ_{ID}'}{dV_D} = -\frac{W}{L}\mu Q_I'(V_D, V_G)$$

⇓

$$dQ_{ID}'\left(\frac{1}{nC_{ox}'} - \frac{\phi_t}{Q_{ID}'}\right) = dV_D$$

depletion charge linearization

charge sheet approximation

The ACM model-5: UCCM

$$dQ'_I\left(\frac{1}{nC'_{ox}}-\frac{\phi_t}{Q'_I}\right)=dV_C \quad \textbf{(I)}$$

Integrating (I) between V_C and V_P

$$\frac{Q'_{IP}-Q'_I}{nC'_{ox}}+\phi_t\ln\left(\frac{Q'_I}{Q'_{IP}}\right)=V_P-V_C$$

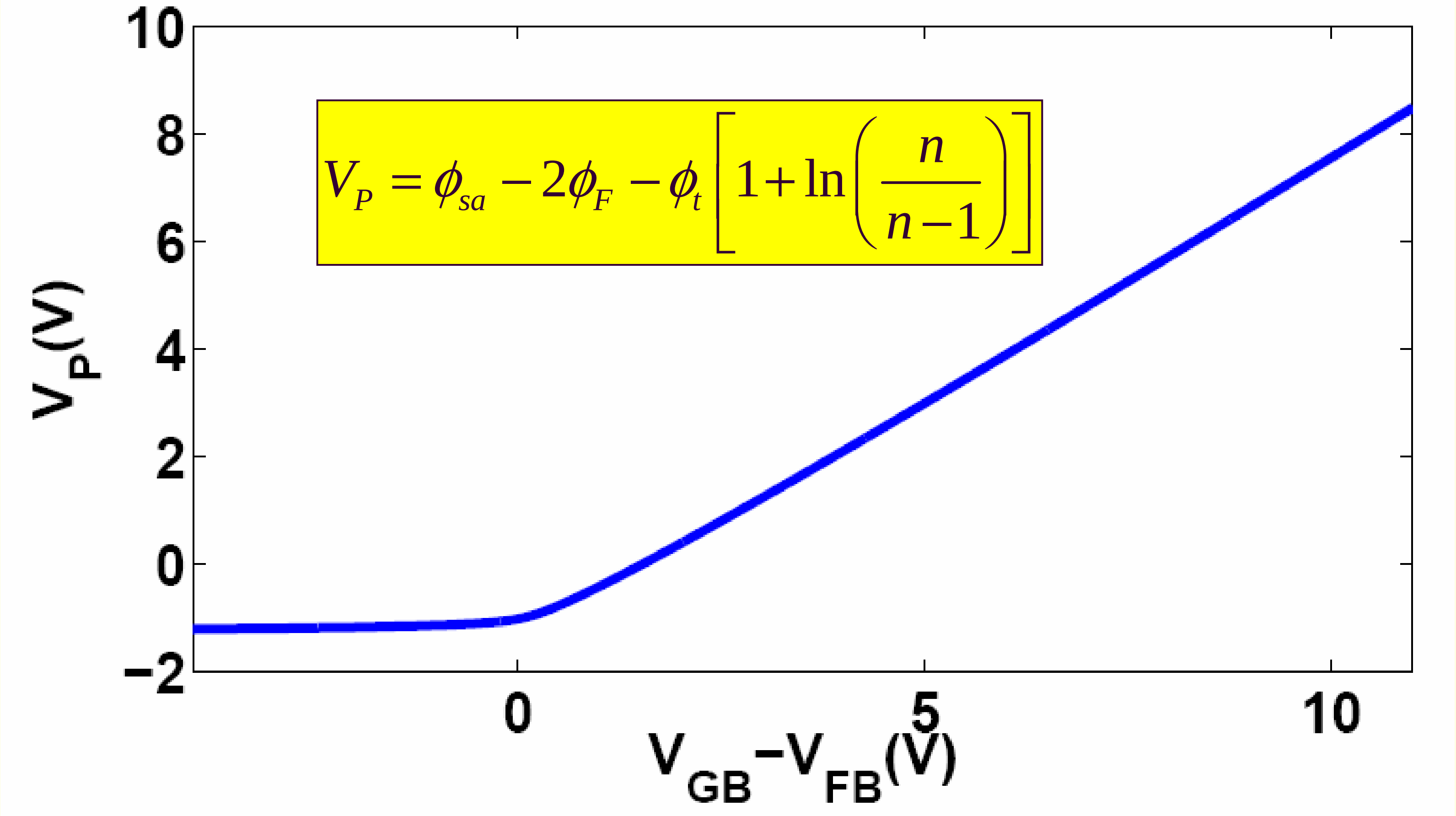
$$\frac{\partial Q'_{IS(D)}}{\partial V_{S(D)}}=nC'_{ox}\frac{Q'_{IS(D)}}{Q'_{IS(D)}-nC'_{ox}\phi_t}$$

$$\frac{\partial Q'_I}{\partial V_G}=-\frac{1}{n}\frac{\partial Q'_I}{\partial V_C}$$

$$\frac{\partial Q'_I}{\partial V_B}=-\frac{n-1}{n}\frac{\partial Q'_I}{\partial V_C}$$

Calculation of the capacitve coefficients

The ACM model-6: The pinch-off voltage $V_P(V_G)$



$$\phi_{sa} = \phi_s\big|_{Q_I'=0}$$

Small-dimension effects on charges and capacitances

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

$$\mu = \frac{\mu_s}{1 + \dfrac{\mu_s}{nC_{ox}' v_{sat}} \dfrac{dQ_I'}{dy}}$$

$$\frac{dQ_I'}{dy}\left(\frac{1}{nC_{ox}'} - \frac{\phi_t}{Q_I'}\right) = \frac{dV_C}{dy}$$

$$dy = -\frac{\mu_s W}{nC_{ox}' I_D}\underbrace{\left(Q_I' - nC_{ox}'\phi_t + \frac{I_D}{W v_{sat}}\right)}_{Q_V'}\underbrace{dQ_I'}_{dQ_V'}$$

Virtual charge Q_V' dQ_V'

Stored charges

The stored charge $$Q_I = W\int_0^{L-\Delta L} Q_I' dy + W\Delta L Q_{IDsat}'$$

is calculated changing the variable from *y* to Q'_V

$$dy = -\frac{\mu_s W}{nC_{ox}' I_D} Q_V' dQ_V'$$

$$Q_I = W(L-\Delta L)\left[\frac{2}{3}\frac{1+\alpha+\alpha^2}{1+\alpha} Q_{VS}' + nC_{ox}'\phi_t\right] - \frac{LI_D}{v_{sat}}$$

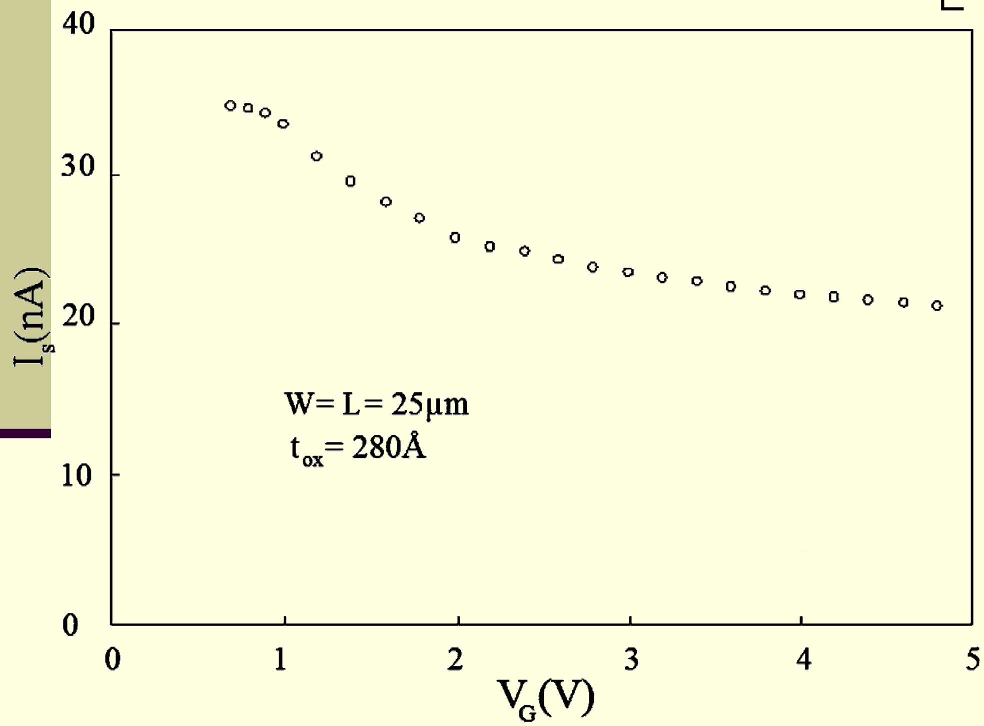
$$\alpha = \frac{Q_{VD}'}{Q_{VS}'}$$

Design-oriented MOSFET model - 1

I_F: forward current I_R: reverse current

$$I_D = I_F - I_R = I_{SQ}\frac{W}{L}\left[i_f - i_r\right]$$

$$V_P - V_{S(D)} = \phi_t\left[\sqrt{1+i_{f(r)}} - 2 + \ln\left(\sqrt{1+i_{f(r)}} - 1\right)\right]$$



UICM

$$I_{SQ} = \mu C'_{ox} n \frac{\phi_t^2}{2}$$

$I_{SQ} \approx$25 nA (p-channel)

$I_{SQ} \approx$75 nA (n-channel)

in 0.35 µm CMOS

Design-oriented MOSFET model - 2

Weak inversion
$i_{f(r)}<1$

$$\frac{V_G - V_{T0}}{n} - V_{S(D)} = \phi_t\left[\underbrace{\sqrt{1+i_{f(r)}} - 2}_{-1} + \ln\Big(\underbrace{\sqrt{1+i_{f(r)}} - 1}_{i_{f(r)}/2}\Big)\right]$$

$$I_D = I_0 e^{\left(\frac{V_G - V_{T0}}{n} - V_S + \phi_t\right)/\phi_t}\left[1 - e^{-V_{DS}/\phi_t}\right] \qquad I_0 = \mu_n \frac{W}{L} n C'_{ox}\phi_t^2 = 2I_S$$

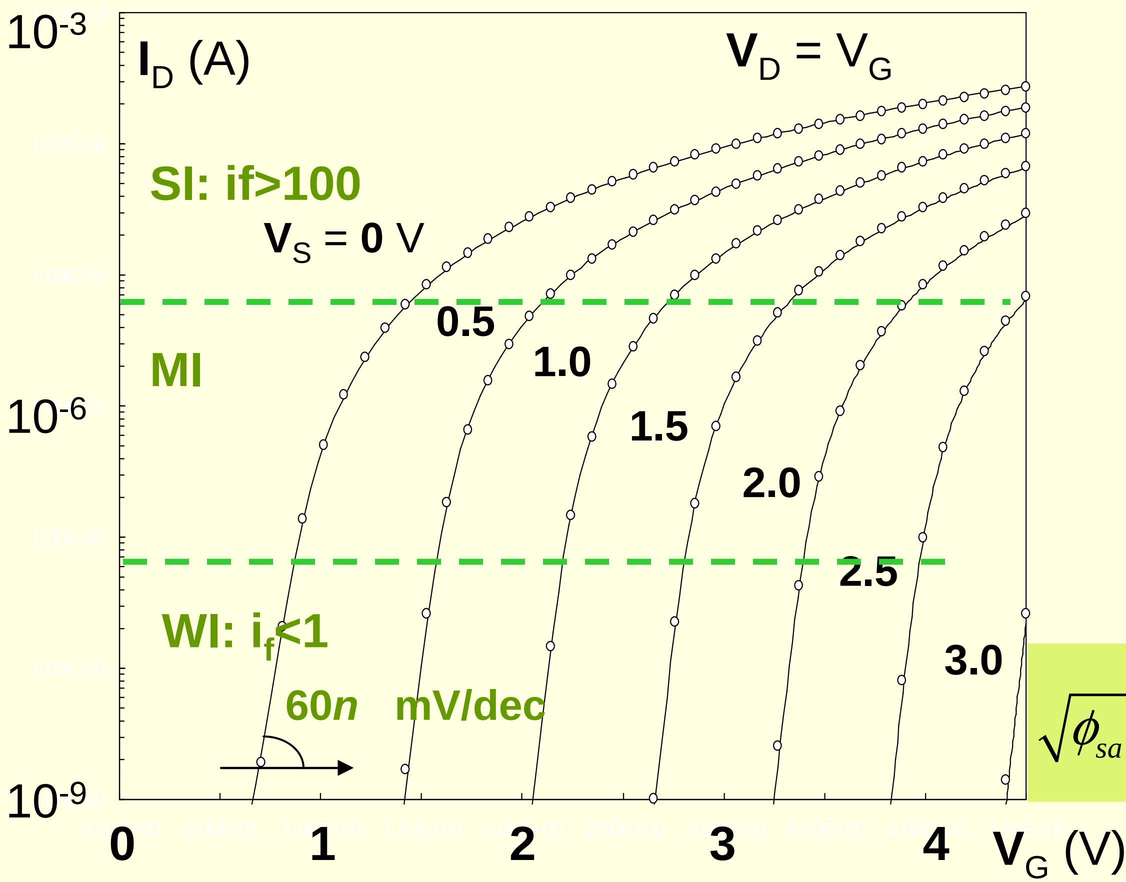
Strong inversion
$i_{f(r)}>>1$

$$\frac{V_G - V_{T0}}{n} - V_{S(D)} \cong \phi_t\sqrt{i_{f(r)}} = \phi_t\sqrt{I_{F(R)}/I_S}$$

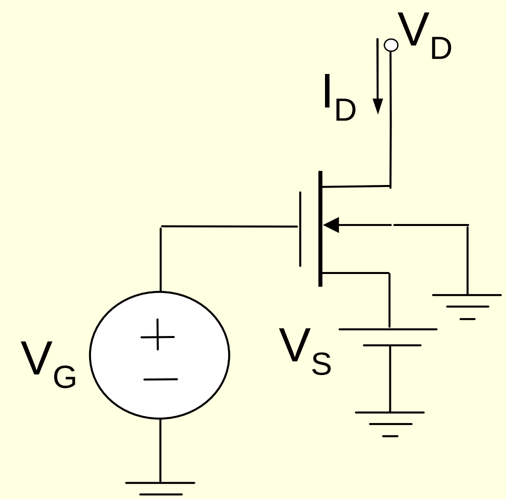
$$I_D = I_F - I_R \cong \mu_n C'_{ox}\frac{W}{2nL}\left[\left(V_G - V_{T0} - nV_S\right)^2 - \left(V_G - V_{T0} - nV_D\right)^2\right]$$

Design-oriented MOSFET model - 3

$$V_P - V_S = \phi_t\left[\sqrt{1+i_f} - 2 + \ln\left(\sqrt{1+i_{f(r)}} - 1\right)\right]$$



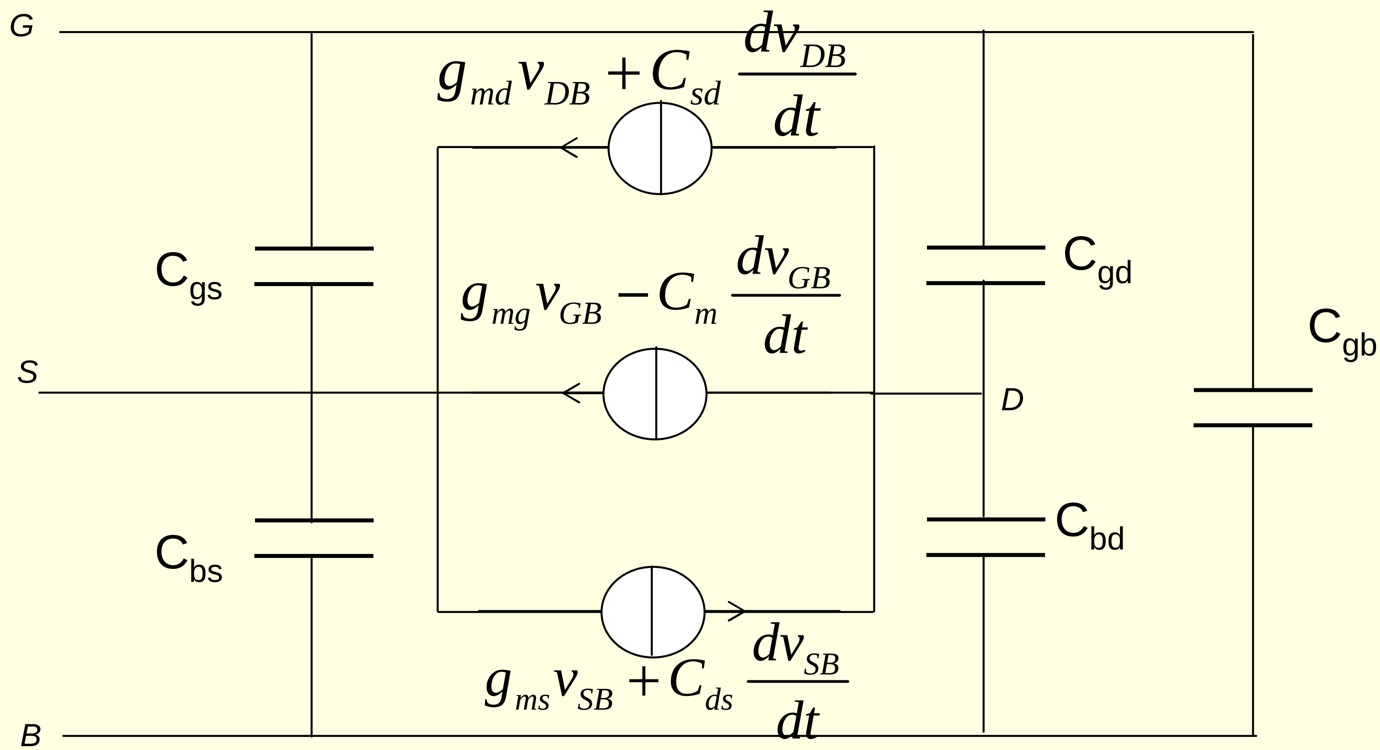
Common-source characteristics



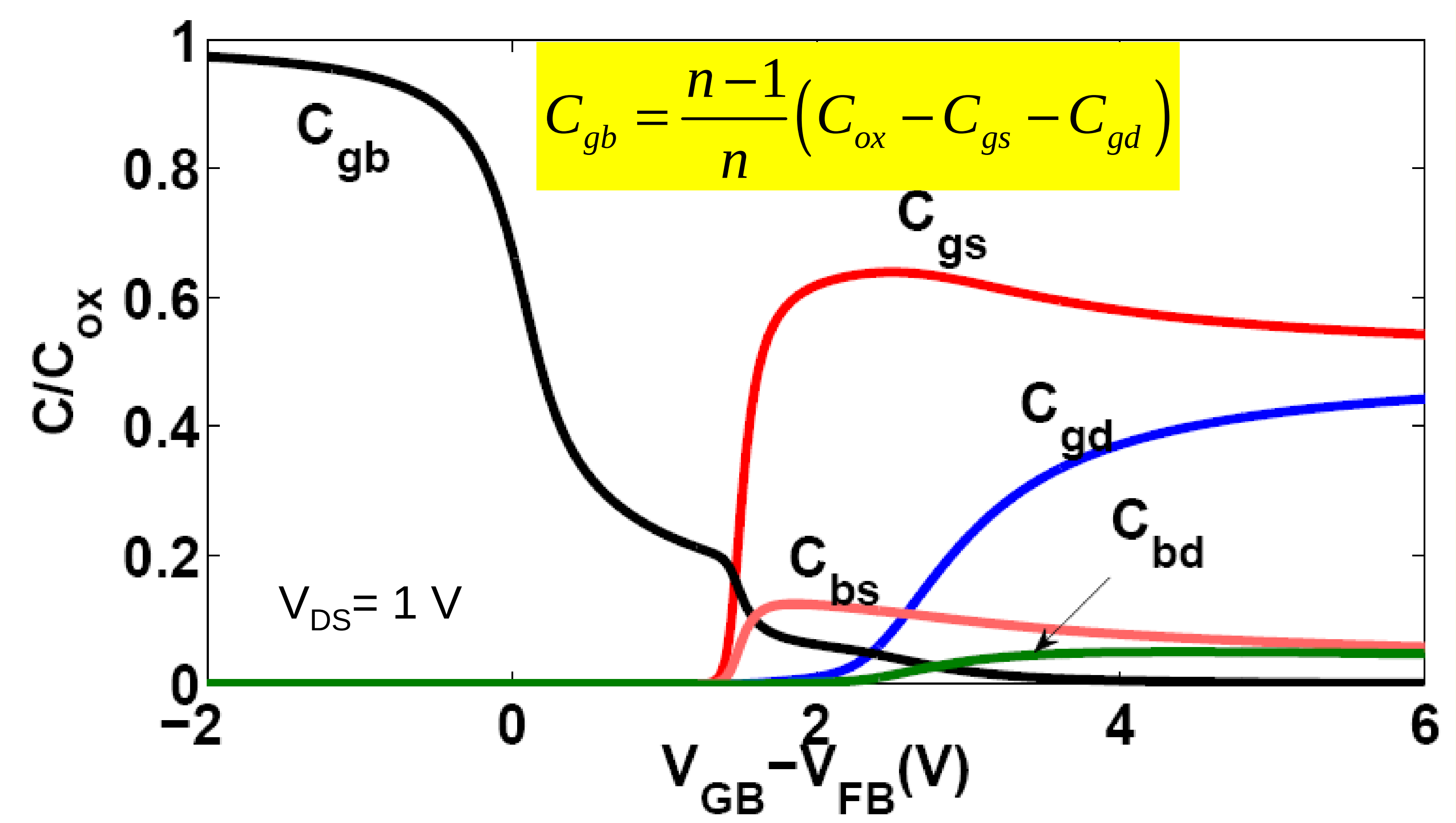
$$V_P \cong \phi_{sa} - \phi_t - 2\phi_F$$

$$\sqrt{\phi_{sa} - \phi_t} \cong \sqrt{V_G - V_{FB} - \phi_t + \frac{\gamma^2}{4}} - \frac{\gamma}{2}$$

Simplified small-signal MOSFET model

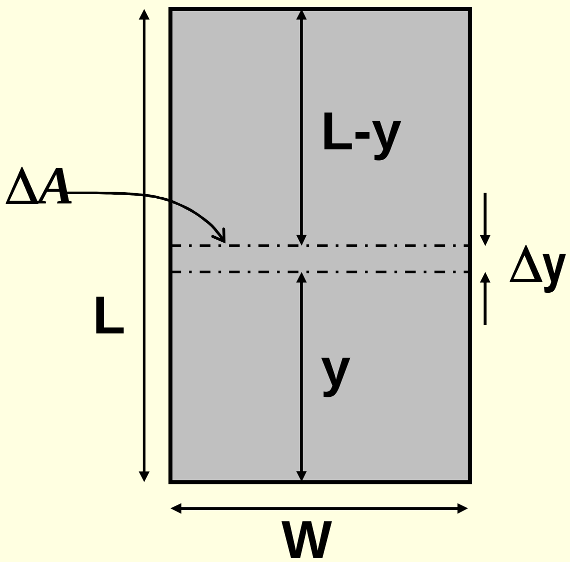


The five capacitances of the simplified MOSFET model



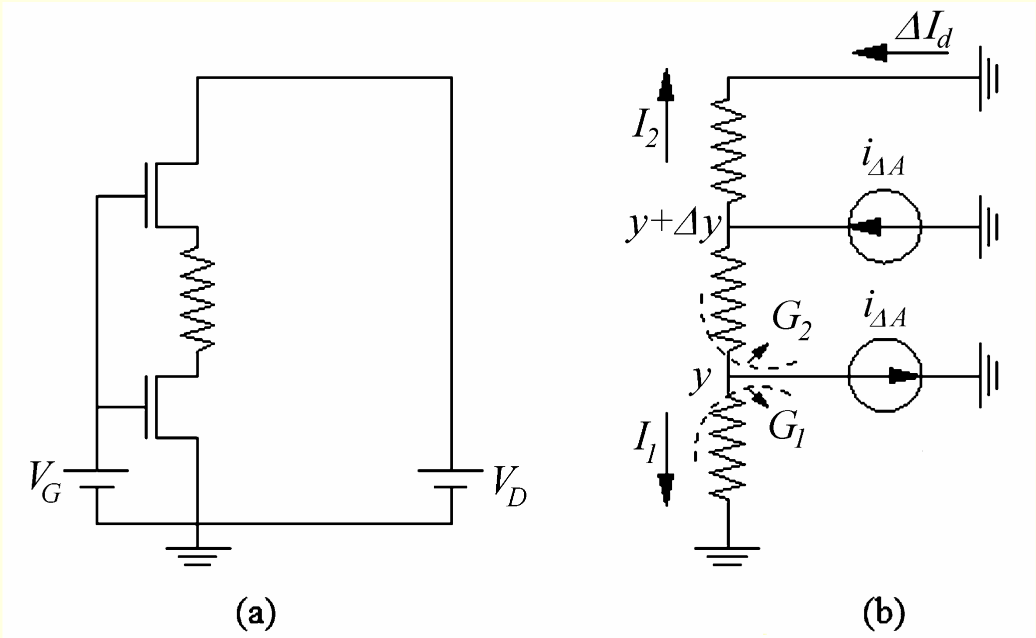
Mismatch and 1/f noise - 1

Channel splitting



Equivalent circuit

Impedance field method



$$G_1 = -\mu \frac{W}{L-y} Q'_{Iy}$$

$$G_2 = -\mu \frac{W}{y} Q'_{Iy}$$

$$\frac{1}{\Delta R} = -\mu \frac{W}{\Delta y} Q'_{Iy}$$

$$\Delta I_d = (\Delta y / L) . i_{\Delta A}$$

Mismatch and 1/f noise - 2

Integration of the small contributions along the channel

$$\frac{\sigma_{I_D}^2}{I_D^2} = \frac{q^2 N_{oi}\mu}{L^2 n C'_{ox} I_D} \ln\left(\frac{nC'_{ox}\phi_t - Q'_{IS}}{nC'_{ox}\phi_t - Q'_{ID}}\right)$$

$$\frac{S_{I_d}}{I_D^2} = \frac{q^2 N_{ot}\mu}{L^2 n C'_{ox} I_D} .\ln\left(\frac{nC'_{ox}\phi_t - Q'_{IS}}{nC'_{ox}\phi_t - Q'_{ID}}\right) .\frac{1}{f}$$

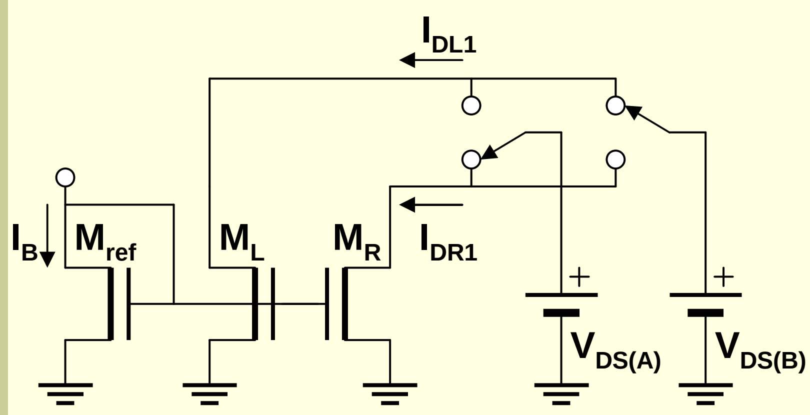
$$\frac{\sigma_{I_D}^2}{I_D^2}\left(f\,\frac{S_{I_d}}{I_D^2}\right) = \frac{N_{oi(t)}}{WLN^{*2}} \frac{1}{i_f - i_r} \ln\left(\frac{1+i_f}{1+i_r}\right)$$

$$N^* = \frac{-Q'_{IP}}{q} = \frac{nC'_{ox}\phi_t}{q}$$

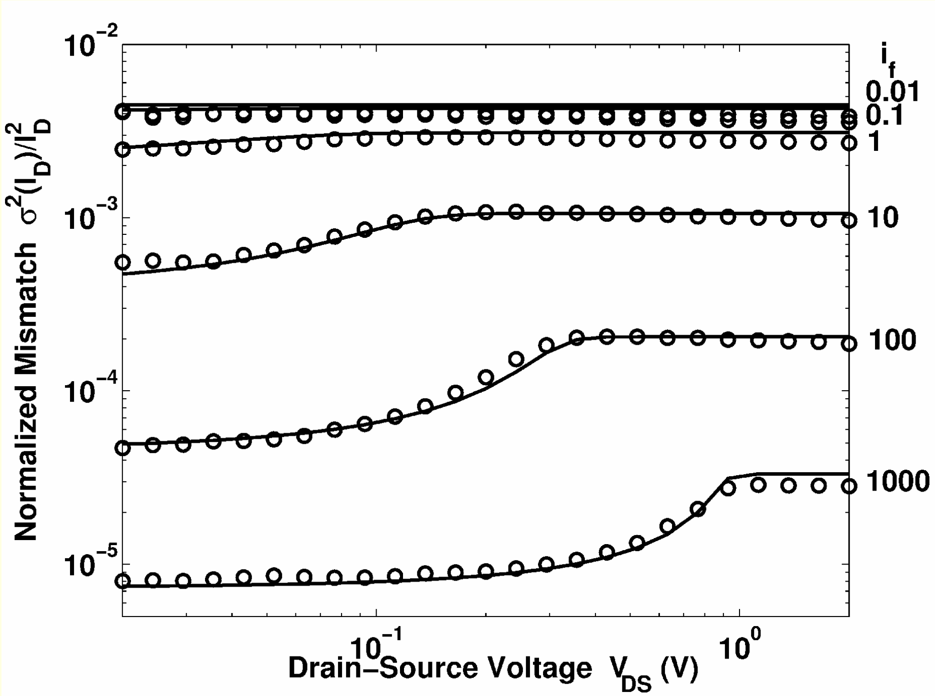
$$I_S = \tfrac{1}{2}\mu C'_{ox} n \phi_t^2 (W/L)$$

Mismatch and 1/f noise - 3

Test circuit



Inversion level dependence



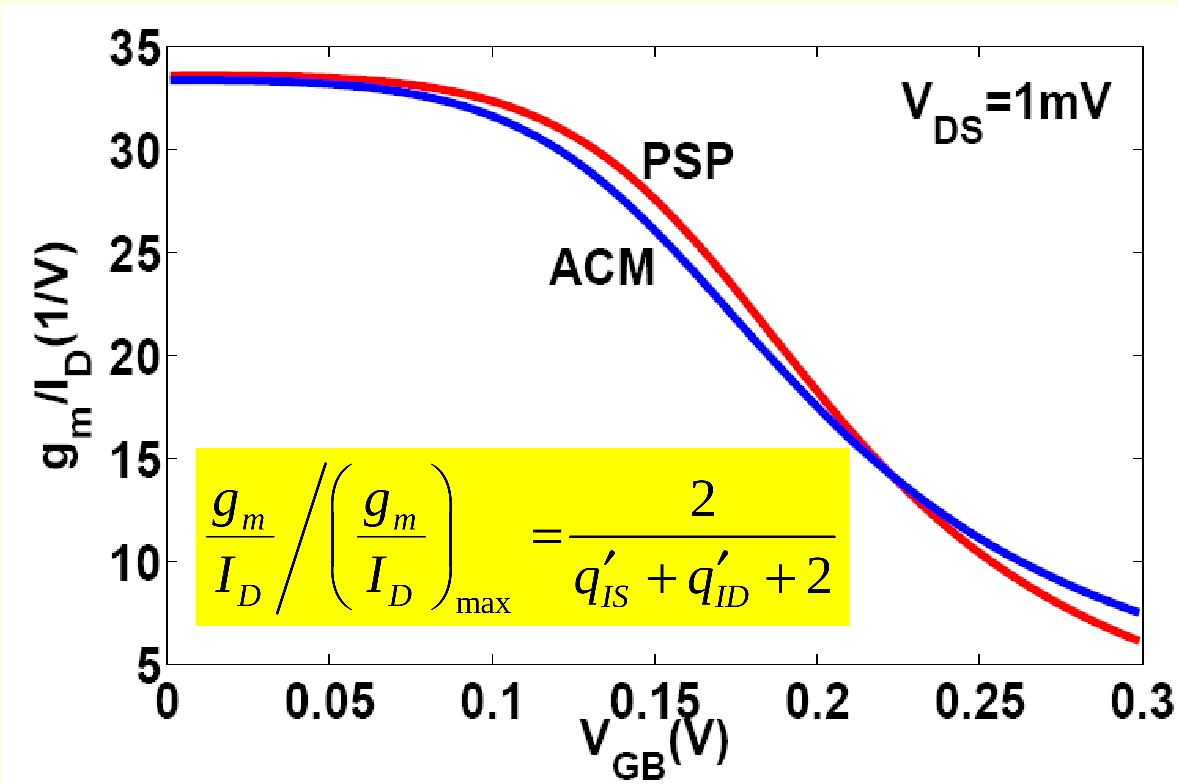
Test chip:

- 9 groups (different geometries)
- 36 N and PMOS pairs per group.

Measured: ooo; Model: —

(TSMC 0.35 NMOS W/L=3μm/2μm)

Parameter extraction

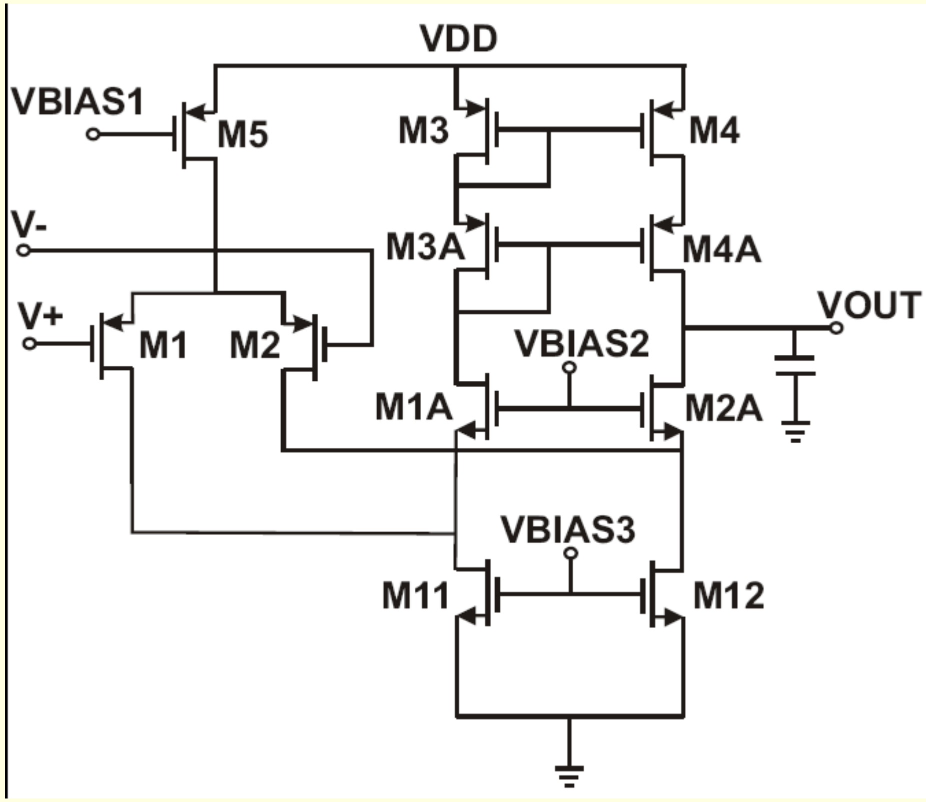


$$(g_m / I_D)_{max} = 1/(n\phi_t)$$

$$q'_{IS(D)} = \frac{Q'_{IS(D)}}{Q'_{IP}} = \frac{Q'_{IS(D)}}{-nC'_{ox}\phi_t}$$

Design example

	Spec	Simulation	Unit
V_{DD}	1.2	1.2	V
C_L	5	5	pF
GBW	10	9.77	MHz
A_{V0}	>100	141	dB
SR	>4	5	V/μs
$V_{ICM, max}$	>V_{DD}-0.3	0.9	V
$V_{ICM, min}$	<0.3	0	V
$V_{O, max}$	>V_{DD}-0.3	0.9	V
$V_{O, min}$	<0.3	0.26	V
Input referred white noise	<100	17.5	nV/Hz$^{1/2}$



Summary: Design oriented expressions for long-channel MOSFET in saturation

i_f	I_D / I_S
I_S	$I_{SQ}\dfrac{W}{L} = \dfrac{\mu C'_{ox} n\phi_t^2}{2}\dfrac{W}{L}$
g_m	$\dfrac{I_D}{n\phi_t}\dfrac{2}{\sqrt{1+i_f}+1}$
V_{DSsat}	$\phi_t\left(\sqrt{1+i_f}+3\right)$
f_t	$\dfrac{\mu\phi_t}{\pi L^2}\left(\sqrt{1+i_f}-1\right)$
$\dfrac{\sigma_{I_D}^2}{I_D^2}\left(f\dfrac{S_{I_D}}{I_D^2}\right)$	$\dfrac{N_{oi(t)}}{WLN^{*2}}\dfrac{\ln\left(1+i_f\right)}{i_f}$
V_P-$V_{S(D)}$	$\phi_t\left(\sqrt{1+i_f}-2+\ln\left(\sqrt{1+i_f}-1\right)\right)$

Conclusions

- **All-region (accumulation, WI, MI and SI) charge-sheet compact MOSFET model fully consistent with the Pao-Sah formula uses the same approximations for the input (electrostatic) and output (transport) equations**

- **dc, ac, ac-nonquasistatic, noise and mismatch design formulas valid in all operating regions**

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